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A New Class of Decomposable Inequality Measures

Discussion paper n. 36/2025

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Keywords: Inequality, Decomposition, Pigou-Dalton Transfer

JEL Classification: D31, D63, O15

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A New Class of Decomposable Inequality Measures

Domenico Moramarco, and Olivier Sterck*

June 2025

Abstract

This paper characterizes the class of inequality measures that are multiplicatively decomposable, meaning they can be expressed as a product of within-group and between-group inequality components, with weights summing to one. Remarkably, this corresponds to the class of inequality measures that is additively decomposable in subgroups, so that that total inequality can be written as the weighted sum of inequalities within groups. The proposed measures satisfy standard axioms in inequality measurement, including scale and population independence, the Pigou-Dalton transfer principle, and—for reasonable parameter values—the transfer sensitivity principle. We illustrate the properties of the new class using data on global income inequality and inequality within the United States.

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1 Introduction

A fundamental question in the analysis of income inequality is the extent to which overall inequality can be attributed to differences between major population subgroups, such as those defined by location, age, sex, race, or education (Bourguignon 1979; Milanovic 2011). Decomposability is a useful property, facilitating subgroup analysis and policy evaluation. However, not all decomposable measures are necessarily desirable indicators of inequality. A robust inequality index should also satisfy fundamental axioms and ethical principles, and have an interpretation that resonates with academics and policy-makers.

The theoretical and empirical literature on decomposable inequality measures has predominantly focused on additive decomposability in between- and within-group contributions, which, under mild assumptions, uniquely characterizes the Generalized Entropy class of inequality measures (Shorrocks 1980).[1] Given a vector of incomes y and a partition of the population into G subgroups, the standard additive decomposition of an inequality index I takes the form

$$I(y) = \underbrace{I(\bar{y})}_{\text{Between-group}} + \underbrace{\sum_{g=1}^{G} w_g I(y^g)}_{\text{Within-group}} , \tag{1}$$

where $\bar{y}$ is the so called *smoothed distribution*, obtained by replacing each income in y by the average of its group, y^g is the vector of incomes within subgroup g, G is the number of subgroups, and w_g is a weight attached to inequality in subgroup g.

Beyond the analytical simplicity of the additive form, a key justification for this focus is the argument that any more general form of decomposability inevitably leads to inequality measures that are mere monotonic transformations of the Generalized Entropy family (Shorrocks 1984). This perceived redundancy has reinforced the focus on additive decomposability in the study of inequality measurement.

Additive decomposability as in equation (1), however, comes with important limita-

[1] Related results were developed by Bourguignon (1979) and Anand (1983).

tions (Shorrocks 1980). First, for most members of the Generalized Entropy class, the decomposition weights $(w_1, ..., w_G)$ do not sum to one. As a result, the within-group component can diverge from the average inequality across groups—even when all subgroups exhibit identical inequality. Second, the sum of decomposition coefficients typically depends on the level of between-group inequality. This implies that even with uniform within-group inequality, changes in between-group inequality can affect the total within-group contribution. Notably, two members of the Generalized Entropy class—the Theil T and L indices—do not suffer from these issues, as their decomposition weights do sum to one.[2] However, Theil indices and other Generalized Entropy measures often remain confined to decomposition analysis, as they lack the intuitive appeal needed for broader use in policy and public debates (Haddad et al. 2024). Sen (1997, p. 36) famously remarked that the Theil index *"is an arbitrary formula, and the average of the logarithms of the reciprocals of income shares weighted by income is not a measure that is exactly overflowing with intuitive sense."* In contrast, measures like the Gini coefficient or quantile ratios (e.g., Palma) are more easily grasped and widely used in policy and public discourse, even though they lack desirable properties in terms of decomposition and distribution sensitivity. As a result, applied research often resorts to a dual approach: intuitive measures like the Gini or quantile ratios are used to describe inequality levels and trends, while the Theil L is brought in specifically for decomposition analysis (e.g., Milanovic 2011; Bourguignon 2015; Ravallion 2018; Milanovic 2024). These limitations raise a natural question: could an alternative form of decomposability yield a class of inequality measures that is both decomposable and intuitively interpretable?

Our paper explores two alternative forms of inequality decomposition. First, we examine the multiplicative decomposability of total inequality into within-group and between-group components.[3] Using the same notation as before, a measure I is multiplicatively

[2] The Theil T and L indices correspond to GE(1) and GE(0), respectively. In the case of the Theil T index, group weights are proportional to each group's total income, reflecting income-weighted decomposability (Bourguignon 1979). This feature gives greater weight to richer groups, which may seem misaligned with the normative view that inequality is more concerning when it affects poorer populations. By contrast, the Theil L index assigns weights proportional to group population sizes, capturing population-weighted decomposability (Bourguignon 1979).

[3] Contrary to this paper, Lasso de la Vega and Urrutia (2008) study the multiplicative decomposability of "equality measures" and characterize a family of generalized Atkinson inequality measures whose

decomposable if

$$I(y) = \underbrace{I(\bar{y})}_{\text{Between-group}} \cdot \underbrace{\sum_{g=1}^{G} \nu_g I(y^g)}_{\text{Within-group}}, \tag{2}$$

for some weights $\nu_1, ..., \nu_G$.[4] Several arguments justify our focus on multiplicative decomposability. First, like the additive decomposition, it is analytically tractable and admits a simple graphical interpretation (see Figure 1). Unlike the additive form, however, it explicitly captures the complementarity between within- and between-group components.[5] Second, multiplicative decompositions can be conveniently reformulated as an addition using a logarithmic transformation. With multiplicative decomposability, percentage changes in total inequality can be expressed as the sum of percentage changes in its within- and between-group components. Multiplicative decomposition is therefore particularly relevant for dynamic analyses of inequality changes.

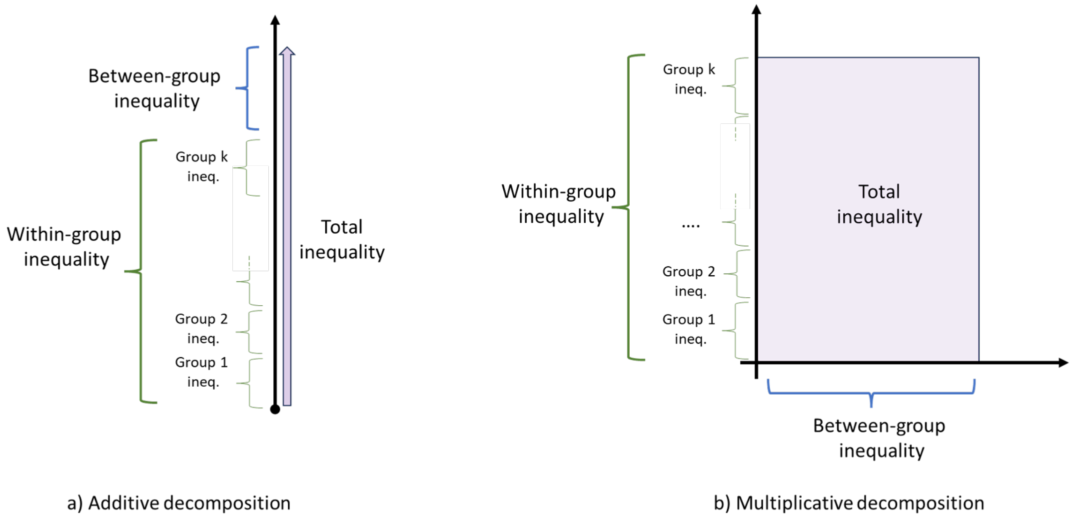


Figure 1: Additive and multiplicative inequality decompositions in within- and between-group components

Second, we also examine the additive decomposition in subgroups, where each group is assigned a direct contribution to total inequality, without distinguishing a separate

corresponding equality measure is multiplicatively decomposable.

[4]An alternative formulation in which each group's within-group contribution and weight enter multiplicatively would be undesirable, as it would imply that the effect on within-group inequality of rising inequality in one group depends on the level of inequality in other groups.

[5]With the Generalized Entropy class, the complementarity is implicitly present—but hidden—as within-group weights generally depend on between-group inequality.

between-group component. An inequality measure I is additively decomposable in subgroups if

$$I(y) = \sum_{g=1}^{G} \omega_g I(y^g) \ , \tag{3}$$

for some weights $\omega_1, ..., \omega_G$. This form of additive decomposition makes it possible to identify which groups contribute most to overall inequality. It is particularly useful when researchers or policymakers seek to assign a specific inequality contribution to each subgroup—for example, to determine which country contributes most to global inequality.

We derive two central results in this paper. First, we establish that multiplicative decomposition as defined in equation (2) is equivalent to additive decomposition in subgroups as defined in equation (3). Second, we characterize the class of inequality measures that satisfy these two properties under relatively weak axiomatic assumptions. The new class takes the form of a single-parameter family given by

$$I_\epsilon(y) = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{y_i}{\mu} \right)^{1-\epsilon} ,$$

where μ is the average income and $\epsilon \in (-\infty, 0) \cup (1, \infty)$ measures inequality aversion. Imposing transfer sensitivity (Shorrocks and Foster 1987) further restricts the range of the inequality aversion parameter to $\epsilon \in (-1, 0) \cup (1, \infty)$.

In the multiplicative decomposition of equation (2), the weights assigned to each group in the within-group component sum to one, ensuring a straightforward interpretation of each group's relative contribution. When $\epsilon > 1$, these weights increase with group population size and decrease with group average income—appropriately assigning greater weight to large and poor economies. In the additive decomposition in subgroups of equation (3), the weights reflect each group's contribution to between-group inequality. In particular, the sum of the weights is itself a measure of inequality between groups. If $\epsilon > 1$, a group's contribution to total inequality is increasing with population size and inequality within the group, and decreasing with the group's average income.

By construction, the new class satisfies key properties in the measurement of inequality, including Scale Invariance, Population Independence, Anonymity, and the Transfer Principle. For reasonable values of parameters, it also satisfies Transfer Sensitivity (Shorrocks and Foster 1987). The new class of inequality measure is directly related to Generalized Entropy and Atkinson (1970) classes of inequality measures (see Section 3 for a discussion). However, a key distinction between the new class and most existing inequality measures lies in its normalization. Since 1 is the identity element of multiplications, perfect equality corresponds to a value of one in our framework.[6]

The measure with an inequality aversion parameter of $\epsilon = 2$ corresponds to the index identified in Kraay et al. (2024). This measure plays a central role within the new class. First, an inequality aversion coefficient of 2 aligns with recent empirical estimates (see e.g., Del Campo et al. 2024; Sterck 2024; Kot and Paradowski 2022). Second, this specific index offers intuitive interpretations. It corresponds to the expected ratio of incomes between two randomly selected individuals in the population (Sterck 2024). The measure can also be interpreted as the average factor by which individual incomes must be multiplied to reach the mean. Third, the inequality measure $I_2(y)$ is directly linked to the prosperity gap (Kraay et al. 2024; Sterck 2024), an inclusive indicator adopted by the World Bank to track progress toward its Shared Prosperity goal.

The remainder of the paper is structured as follows. Section 2 adopts an axiomatic approach to characterize the class of inequality measures satisfying the decomposition properties in equations (2) and (3). Section 3 examines the key properties of this class, with particular emphasis on the measure corresponding to an inequality aversion parameter of 2. Section 4 illustrates the measure and its decomposition properties, considering both global inequality and inequality in the US. Section 5 concludes.

[6]The identity element of the addition is 0. This is the minimum value of inequality measures like the Gini or the members of the Generalized Entropy class. For our class of measures, 1 is the minimum value possible.

2 Theoretical Framework

2.1 Preliminaries

Let $y = (y_1, ..., y_n) \in \mathbb{R}^n_{++}$ be an income distribution that can be partitioned in $G \geq 2$ groups, so that $y = \left(y^1, y^2, ..., y^G\right)$, with $y^g = \left(y^g_1, ..., y^g_{n_g}\right) \in \mathbb{R}^{n_g}_{++}$ for all $1 \leq g \leq G$.[7] Let $\mu \in \mathbb{R}_{++}$ denote the average income, and $\boldsymbol{\mu} = (\mu_1, ..., \mu_G) \in \mathbb{R}^G_{++}$ and $\boldsymbol{n} = (n_1, ..., n_G) \in \mathbb{N}^G_{++}$ the vectors of, respectively, groups' mean and population size. We assume that $n_g \geq 2$ for all $1 \leq g \leq G$. We denote by $\bar{y} \in \mathbb{R}^n_{++}$ the smoothed distribution which is obtained by replacing each income y_i by the average income of the group it belongs to.

Let $I : \mathbb{R}^n_{++} \to \mathbb{R}_+$ be an inequality measure. For any $y \in \mathbb{R}^n_{++}$, standard desirable properties for I are (see, for example, Villar 2017, ch. 2):

Axiom 1. Continuity: $I(y)$ is continuous with continuous first-order partial derivatives.

Axiom 2. Symmetry: $I(y) = I(\Pi y)$ for any permutation matrix Π.

Axiom 3. Transfer principle: $I(y) > I(By)$ for any bistochastic matrix B that is neither the identity nor a permutation matrix.

Axiom 4. Scale Invariance: $I(\lambda y) = I(y)$ for all $\lambda \geq 1$.

Axiom 5. Replication Invariance: $I\left(\underbrace{y, ..., y}_{\times m}\right) = I(y)$ for all $m \in \mathbb{N}_{++}$.

Throughout the paper, a measure I is an inequality measure only if it satisfies the above properties. This convention allows us to avoid mentioning the above axioms in each of the following results.

We enlarge the set of desirable properties for I by including the two decomposability requirements discussed in the previous section:

Axiom 6. Multiplicative decomposability: For all $y \in \mathbb{R}^n_{++}$, there exists a list of coefficients $\nu_g(\boldsymbol{\mu}, \boldsymbol{n})$, for $g = \{1, ..., G\}$, such that

$$I(y) = I(\bar{y}) \left[\sum_{g=1}^{G} \nu_g(\boldsymbol{\mu}, \boldsymbol{n}) I(y^g)\right]. \tag{4}$$

[7] Throughout the text $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}_{++}$ the set of strictly positive real numbers, and $\mathbb{R}_+ = \mathbb{R}_{++} \cup \{0\}$.

Axiom 7. Additive decomposability in subgroups: For all $y \in \mathbb{R}^n_{++}$, there exists a list of coefficients $\omega_g(\boldsymbol{\mu}, \boldsymbol{n})$, for $g = \{1, ..., G\}$, such that

$$I(y) = \sum_{g=1}^{G} \omega_g(\boldsymbol{\mu}, \boldsymbol{n}) I(y^g). \tag{5}$$

Our first result is to show that these two axioms are actually equivalent. To see this, observe that $(\boldsymbol{\mu}, \boldsymbol{n})$ is sufficient information for constructing $\bar{y}$. Therefore, by defining $\omega_g(\boldsymbol{\mu}, \boldsymbol{n}) = I(\bar{y})\ \nu_g(\boldsymbol{\mu}, \boldsymbol{n})$, any multiplicative decomposition can be written as additive decomposition in subgroups, and vice versa. Formally:

Theorem 1. *An inequality measure I satisfies Multiplicative decomposability if and only if it satisfies Additive decomposability in subgroups.*

Proof. Suppose that I satisfies Multiplicative decomposability. Then, for any $y \in \mathbb{R}^n_{++}$, there exist numbers $\nu_g(\boldsymbol{\mu}, \boldsymbol{n})$, $1 \leq g \leq G$, such that

$$I(y) = I(\bar{y}) \left[\sum_{g=1}^{G} \nu_g(\boldsymbol{\mu}, \boldsymbol{n}) I(y^g)\right].$$

Define $x = (\mu_1 \mathbf{1}_{n_1}, ..., \mu_G \mathbf{1}_{n_G})$, where $\mathbf{1}_k$ is the k-dimensional unit vector. By Symmetry, $I(x) = I(\bar{y})$. Hence, $I(\bar{y})$ is function of $(\boldsymbol{\mu}, \boldsymbol{n})$.

Let $\omega_g(\boldsymbol{\mu}, \boldsymbol{n}) \equiv I(\bar{y})\ \nu_g(\boldsymbol{\mu}, \boldsymbol{n})$ for all $1 \leq g \leq G$. Substituting in the previous equation we obtain

$$I(y) = \sum_{g=1}^{G} \omega_g(\boldsymbol{\mu}, \boldsymbol{n}) I(y^g)$$

which is the additive decomposition in subgroups.

For the other direction of the implication, it is sufficient to notice that if I exists, then we can compute $I(\bar{y})$ and apply the reverse reasoning. $\square$

The last minimal requirement for I is a normalization. In other words, there should exist $K \in \mathbb{R}_+$ such that $I(y) \geq K$ with $I(y) = K$ if and only if $y_i = \mu$ for all $1 \leq i \leq n$. The reader may notice that it is common to set $K = 0$. The following lemma shows that 0 cannot be the value corresponding to perfect equality.

Lemma 1. *If the inequality measure I satisfies Multiplicative decomposability, then we cannot impose $I(y) = 0$ if and only if $y_i = \mu$ for all $1 \leq i \leq n$.*

Proof. Consider a distribution y such that $\mu_g = \mu$ for all $1 \leq g \leq G$. By multiplicative separability, $I(y) = \left(\sum_{g=1}^{G} \omega_g(\mu, \boldsymbol{n}) I(y^g)\right) I(\bar{y})$. Now, since $I(\bar{y}) = 0$, then $I(y) = 0$ for all $I(y^1), ..., I\left(y^G\right)$. This contradicts the Transfer Principle. □

Since one is the neutral element of a multiplication, and given our focus on multiplicative decomposability, we find it desirable to impose the following normalization.[8]

Axiom 8. Normalization: $I(y) \geq 1$ with $I(y) = 1$ if and only if $y_i = \mu$ for all $1 \leq i \leq n$.

The following lemma clarifies the implication of our normalization on the weights attached to each within-group inequality measure. Normalization in which perfect equality corresponds to 1 leads decomposition weights summing to 1 (while a normalization to a constant $K > 0$ would lead to decomposition weights summing to $1/K$).

Lemma 2. *Let I be an inequality measure that satisfies Normalization. If I satisfies Multiplicative decomposability, then equation (4) must hold with $\sum_{g=1}^{G} \nu_g(\boldsymbol{\mu}, \boldsymbol{n}) = 1$.*

Proof. By way of contradiction, assume that $\sum_{g=1}^{G} \nu_g(\boldsymbol{\mu}, \boldsymbol{n}) = K \neq 1$. Consider a distribution y divided in groups of heterogeneous size and average, but such that each income within a group corresponds to its average. In other words, $y = \left(\bar{y}^1, ..., \bar{y}^G\right) = \bar{y}$ for some $(\boldsymbol{\mu}, \boldsymbol{n})$. If I satisfies Multiplicative decomposability then it must be

$$I(y) = I\left(\bar{y}^1, ..., \bar{y}^G\right) = \left(\sum_{g=1}^{G} \nu_g(\boldsymbol{\mu}, \boldsymbol{n}) I(\bar{y}^g)\right) I(y).$$

By Normalization, $I(\bar{y}^g) = 1$ for all g. Therefore,

$$I(y) = \left(\sum_{g=1}^{G} \nu_g(\boldsymbol{\mu}, \boldsymbol{n})\right) I(y) = K\ I(y).$$

A contradiction. □

[8] Lasso de la Vega and Urrutia (2008) imposed a normalization in which the perfect equality corresponds to zero, and thus were unable to identify a class of multiplicatively decomposable inequality measures. Instead, they derived a class of multiplicatively decomposable equality measures.

Hence, in a multiplicative decomposition, the weights assigned to each within-group inequality measure must sum up to 1. This has an interesting implication for the weights in the group decomposition. Consider a distribution y with no inequality within groups, so that $I(y) = I(\bar{y})$ and $I(y^g) = 1$ for all groups. Substituting in equation (5), we get

$$I(\bar{y}) = \sum_{g=1}^{G} \omega_g\left(\boldsymbol{\mu}, \boldsymbol{n}\right). \tag{6}$$

In other words, the sum of the weights in the additive decomposition in subgroups must correspond to a measure of between-group inequality. A consequence of Normalization is that, differently from the weights in the multiplicative decomposition, $\sum_{g=1}^{G} \omega_g\left(\boldsymbol{\mu}, \boldsymbol{n}\right) \geq 1$.

2.2 Characterization

The following theorem characterizes the family of inequality measures that are additively decomposable in subgroups. Given Theorem 1, it also characterizes the family of inequality measures that are multiplicatively decomposable in within- and between-group components.

Theorem 2. *The function I is an inequality measure satisfying Normalization and Additive decomposability in subgroups if and only if there exists $\epsilon \in (-\infty, 0) \cup (1, \infty)$ such that, for all $y \in \mathbb{R}^n_{++}$,*

$$I(y) = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{y_i}{\mu}\right)^{1-\epsilon}. \tag{7}$$

Proof. Let I be an inequality measure satisfying Normalization and Additive decomposability in subgroups.

We begin the proof by introducing some preliminary results. For the most part, they are adaptations of theorems proved in Shorrocks (1980) to our different Normalization axiom. We list them as lemmas and provide the proofs in Appendix A for the sake of completeness. The first lemma of the proof defines a functional form for the the weights in equation (5).

Lemma 3. *If I satisfies Continuity, Symmetry, Normalization and Additive decompos-*

ability in subgroups, then there exists a set of functions $\theta(\mu, n)$ such that

$$\omega_g(\boldsymbol{\mu}, \boldsymbol{n}) = \frac{\theta(\mu_g, n_g)}{\theta(\mu, n)}. \tag{8}$$

Lemma 3 replicates Theorem 1 in Shorrocks (1980). The next lemma shows that Additive decomposability in subgroups leads to an additive form. The reader may observe the similarity between the second term on the right hand side of equation (9) and equation (15) in Shorrocks (1980).

Lemma 4. *I satisfies Continuity, Symmetry, Normalization, Transfer Principle and Additive decomposability in subgroups if and only if*

$$I(y) = 1 + \frac{1}{\theta(\mu, n)} \sum_{i=1}^{n} [\phi(y_i) - \phi(\mu)] \tag{9}$$

where $\theta(\mu, n)$ is positive $\theta_\mu(\mu, n)$ and $\phi'(\mu)$ are continuous and ϕ is strictly convex.

The next lemma is also shown in Shorrocks (1980), and in Appendix A.

Lemma 5. *Indices of the form given in (9) satisfy Replication invariance if and only if $\theta(\mu, n) = n\alpha(\mu)$ where $\alpha(\cdot)$ is positive and differentiable.*

We are now endowed with the necessary results to prove the main theorem. It follows from Lemma 5 that

$$\omega_g(\boldsymbol{\mu}, \boldsymbol{n}) = \frac{\theta(\mu_g, n_g)}{\theta(\mu, n)} = \frac{n_g\alpha(\mu_g)}{n\alpha(\mu)}$$

for some positive and continuously differentiable $\alpha(\cdot)$. Combining this with (9) gives us

$$I(y) = 1 + \frac{1}{n\alpha(\mu)} \sum_{i=1}^{n} [\phi(y_i) - \phi(\mu)]$$

where $\theta(\mu, n)$ is positive $\theta_\mu(\mu, n)$ and $\phi'(\mu)$ are continuous and ϕ is strictly convex. We can thus set $\alpha = \phi$ and normalize $\phi(1) = 1$ to get

$$I(y) = 1 + \frac{1}{n\phi(\mu)} \sum_{i=1}^{n} [\phi(y_i) - \phi(\mu)].$$

By Scale Invariance

$$I(y) = I\left(\frac{1}{\mu}y\right) = 1 + \frac{1}{n\phi(1)}\sum_{i=1}^{n}\left[\phi\left(\frac{y_i}{\mu}\right) - \phi(1)\right]$$

$$I(y) = 1 + \frac{1}{n}\sum_{i=1}^{n}\left[\phi\left(\frac{y_i}{\mu}\right) - 1\right]$$

$$I(y) = \frac{1}{n}\sum_{i=1}^{n}\phi\left(\frac{y_i}{\mu}\right) \tag{10}$$

for some strictly convex function ϕ such that $\phi(1) = 1$.

Now let us recall Additive decomposability in subgroups, which implies

$$I(y) = \sum_{g=1}^{G}\frac{n_g\phi(\mu_g)}{n\phi(\mu)}I(y^g).$$

Thus, it must be that

$$\begin{aligned} \tfrac{1}{n}\textstyle\sum_{i=1}^{n}\phi\left(\frac{y_i}{\mu}\right) &= \tfrac{1}{n}\textstyle\sum_{g=1}^{G}\frac{n_g\phi(\mu_g)}{\phi(\mu)}I(y^g) \\ &= \tfrac{1}{n}\textstyle\sum_{g=1}^{G}\frac{n_g\phi(\mu_g)}{\phi(\mu)}\frac{1}{n_g}\sum_{j=1}^{n_g}\phi\left(\frac{y_j^g}{\mu_g}\right) \\ &= \tfrac{1}{n}\textstyle\sum_{g=1}^{G}\frac{\phi(\mu_g)}{\phi(\mu)}\sum_{j=1}^{n_g}\phi\left(\frac{y_j^g}{\mu_g}\right) \end{aligned} \tag{11}$$

We need to find the function ϕ that satisfies the above equality. The function $\phi(x) = x^\alpha$ works. Indeed

$$\frac{1}{n}\sum_{g=1}^{G}\left(\frac{\mu_g}{\mu}\right)^\alpha\sum_{j=1}^{n_g}\left(\frac{y_j^g}{\mu_g}\right)^\alpha = \frac{1}{n}\sum_{g=1}^{G}\left(\frac{1}{\mu}\right)^\alpha\sum_{j=1}^{n_g}\left(\frac{y_j^g}{1}\right)^\alpha = \frac{1}{n}\sum_{g=1}^{G}\sum_{j=1}^{n_g}\left(\frac{y_j^g}{\mu}\right)^\alpha = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\mu}\right)^\alpha.$$

We argue that this solution is unique. To see this, notice that (11) is equivalent to

$$\sum_{g=1}^{G}\sum_{j=1}^{n_g}\phi\left(\frac{y_j^g}{\mu}\right) = \sum_{g=1}^{G}\frac{\phi(\mu_g)}{\phi(\mu)}\sum_{j=1}^{n_g}\phi\left(\frac{y_j^g}{\mu_g}\right).$$

Since μ and μ_g are averages, we can vary them freely. Look at a single term on each side for group g. When the identity must hold for all possible values, it forces, in effect,

$$\phi\left(\frac{x}{\mu}\right) = \frac{\phi(\mu_g)}{\phi(\mu)}\phi\left(\frac{x}{\mu_g}\right) \tag{12}$$

for all μ, $\mu_g > 0$ and $x \in \mathbb{R}_+$, subject to $x/\mu = (x/\mu_g) \cdot (\mu_g/\mu)$. Take the case of $\mu = 1$ and recall that $\phi(1) = 1$. Equation 12 becomes

$$\phi(x) = \frac{\phi(\mu_g)}{\phi(1)}\phi\left(\frac{x}{\mu_g}\right) \iff \phi(x) = \phi(\mu_g)\,\phi\left(\frac{x}{\mu_g}\right).$$

Once again, this must holt for all x and μ_g. Setting $\mu_g = a$ and $x = az$ for some a, z we get

$$\phi(az) = \phi(a)\,\phi(z).$$

Aczél (1966, p. 39 equation 7) shows that the most general solution for the above equation is of the form $\phi(z) = z^{\alpha}$. Replacing, and setting $\alpha = (1 - \epsilon)$ into (10) gives the desired functional form.

We only need to establish the admissible values for ϵ in equation (7). By the Transfer Principle, $(y_i/\mu)^{\alpha}$ ought to be strictly convex for all y_i. Taking the second derivative of the function $f(t) = t^{\alpha}$ we have $f''(t) = \alpha(\alpha-1)t^{\alpha-2}$, which is strictly positive if and only if $\alpha(\alpha - 1) > 0$. This leads to the desired condition $\epsilon < 0$ or $\epsilon > 1$.

This concludes the proof of the necessity. Sufficiency is easy to show and left to the reader. □

In combination with Theorem 1, Theorem 2 identifies a single family of inequality measures that satisfies both of our decomposability axioms. Before turning to a more detailed analysis of the properties of the proposed measures, it is useful to underline that the same family can be restricted to satisfy also the transfer sensitivity principle (Shorrocks and Foster 1987).

Axiom 9. Transfer Sensitivity: A progressive transfer among low incomes reduces I more than an equal transfer taking place among higher incomes.

Lemma 6. *Inequality measures in equation (7) satisfy Transfer Sensitivity if and only if*

$\epsilon \in (-1,0) \cup (1,\infty)$.

Proof. Shorrocks and Foster (1987) show that, when I is differentiable, Transfer Sensitivity corresponds to negative third derivative. Consider each single element of the sum in equation (7). Define, $f(x) = x^{1-\epsilon}$, for $x > 0$, then $f'''(x) < 0$ if and only if $-\epsilon(1-\epsilon)(-\epsilon-1) < 0$. This inequality is satisfied only for $\epsilon \in (-1,0) \cup (1,\infty)$.

□

3 Properties

The previous section demonstrated that the new class of inequality measures defined in equation (7) satisfies a set of axioms widely regarded as fundamental in the measurement of inequality. These include Continuity, Symmetry, Transfer, Replication Invariance, and Scale Invariance. This class is uniquely characterized by its compliance with these axioms, along with the decomposability properties in equations (4) and (5).

The multiplicative decomposition of equation (7) is:[9]

$$I_\epsilon(y) = I_\epsilon(\bar{y}) \sum_{g=1}^{G} \nu_g \; I_\epsilon(y^g), \qquad \text{where} \;\; \nu_g = \frac{n_g \mu_g^{1-\epsilon}}{\sum_{g=1}^{G} n_g \mu_g^{1-\epsilon}}. \tag{13}$$

It is straightforward that the weights ν_g sum to one for any value of the inequality aversion parameter. When $\epsilon > 1$, a group's weight increases with its population size and decreases with its mean income, reflecting a normative emphasis on larger and poorer groups. Greater inequality aversion further amplifies the weight assigned to poorer groups.

The additive decomposition in subgroups of equation (7) is:

$$I_\epsilon(y) = \sum_{g=1}^{G} \omega_g \; I_\epsilon(y^g), \qquad \text{with} \;\; \omega_g = \frac{n_g}{n} \frac{\mu_g^{1-\epsilon}}{\mu^{1-\epsilon}}. \tag{14}$$

In this case, the weights ω_g reflect each group's contribution to between-group inequality—they also increase with a group's population size and decrease with its mean income when $\epsilon > 1$. Particularly, setting all $I_\epsilon(y^g)$ to 1, the previous equation becomes a measure of

[9] We denote I_x the version of equation (7) in which $\epsilon = x$.

inequality between groups, in line with equation (6).

There is a direct link between our results and the work of Shorrocks (1984), which characterizes the class of decomposable inequality measures satisfying a general decomposition property, defined by an aggregator function A such that

$$I\left(y^1, ..., y^G\right) = A\left(I\left(y^1\right), ..., I\left(y^G\right), \boldsymbol{\mu}, \boldsymbol{n}\right). \tag{15}$$

Assuming normalization at 0, Shorrocks (1984)'s Theorem 5 shows that any decomposable inequality measure satisfying equation (15) must be a continuous and strictly increasing transformation $F(I)$ of the Generalized Entropy (GE) class with $F(0) = 0$. Lemma 1 implies normalization at 0 is incompatible with multiplicative decomposability while Lemma 2 shows normalization at 1 is the natural choice for multiplicative decomposition. Since we use normalization at 1 in our paper, Shorrocks (1984)'s result is not directly applicable. Yet, there is a direct link between our new class and the GE class (and hence also with Atkinson (1970)'s class, whose link with the GE class is well known). Specifically, we have $I_\epsilon(y) = -\epsilon\ (1-\epsilon)\ GE_{1-\epsilon}(y) + 1 = [1 - A_\epsilon(y)]^{1-\epsilon}$.[10]

There is also a direct relationship between the new class of inequality measures and the class of inclusive poverty measures introduced by Sterck (2024).[11] It is straightforward that

$$I_\epsilon(y) = P_\epsilon(y; z) \times \frac{z^{1-\epsilon}}{\mu^{1-\epsilon}} = \frac{P_\epsilon(y; z)}{P_\epsilon(\mu \mathbf{1}_n; z)}, \tag{16}$$

where $P_\epsilon(y; z) = \frac{1}{n}\sum_{i=1}^n \left(\frac{y_i}{z}\right)^{1-\epsilon}$ is obtained by replacing μ by a constant z in equation (7). $P_\epsilon(y; z)$ is the class of poverty measures corresponding to the new class of inequality measures (Sen 1976, p.225). For $\epsilon > 1$, $P_\epsilon(y; z)$ is an inclusive and distribution sensitive

[10] The GE class is typically defined as $GE_c(y) = \frac{1}{n}\frac{1}{c(c-1)}\sum_{i=1}^n \left[\left(\frac{y_i}{\mu}\right)^c - 1\right]$ for $c \neq 0, 1$. The Atkinson (1970) class is typically defined as $A_\epsilon(y) = 1 - \frac{1}{\mu}\left(\frac{1}{n}\sum_{i=1}^n y_i^{1-\epsilon}\right)^{1/(1-\epsilon)}$ for $0 \leq \epsilon \neq 1$.

[11] Following Sterck (2024), poverty measures decrease with incomes while prosperity measures increase with income. Poverty (resp. Prosperity) measures are *focused* if they only consider incomes below (resp. above) a poverty (resp. prosperity) line, and inclusive if they consider the entire distribution of incomes. We follow this terminology in this paper.

poverty measure, satisfying both transfer and transfer sensitivity axioms (Kakwani 1980; Foster and Shorrocks 1988).

The measure with an inequality aversion parameter of $\epsilon = 2$ plays a central role in the new class. It corresponds to the inequality index identified in Kraay et al. (2024) and Sterck (2024):

$$I_2(y) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\mu}{y_i}\right) \tag{17}$$

In line with Sterck (2024), we use the label *Average Inequality Ratio* or *Average Inequality* in short, to reflect the fact that it is a simple average of individual inequality functions, which take the form of a ratio between average income and individual income.[12]

This measure has several advantages. First, $\epsilon = 2$ is relatively consistent with recent empirical estimates of inequality aversion. Drawing on an online survey experiment conducted with both experts and members of the general public in the US, South Africa, India, and Kenya, Sterck (2024) reports an average inequality aversion parameter of 2.11 among experts (95% CI: 1.93–2.30, median = 2) and 2.41 in the general public (95% CI: 2.35–2.47, median = 2.75). These findings align closely with values derived from macroeconomic calibration methods (Kot and Paradowski 2022). Estimates derived from taxation data tend to fall between 1 and 2 (Del Campo et al. 2024). The axiomatic literature, which derives the level of inequality aversion required to satisfy core equity principles, typically points to values above 1 or 2 (Fleurbaey and Michel 2001; Del Campo et al. 2024). In the context of pro-poor growth, Foster and Székely (2008) similarly argue that ϵ should be at least 2 to ensure sufficient sensitivity to the lower end of the income distribution.

Second, this specific inequality measure offers intuitive interpretations. It corresponds to the expected ratio of incomes between two randomly selected individuals in the population. For example, in the United States, the measure equals 4.9, meaning that the expected income ratio of two people chosen at random is 4.9. By contrast, the corresponding ratio is only 1.5 in China and 1.4 in India, indicating lower income inequality

[12] Kraay et al. (2024) use the term *mean ratio deviation*.

(Table 1). Alternatively, the measure can be understood as the average factor by which individual incomes must be multiplied to reach the mean. In the US, incomes would need to be multiplied by 4.9 on average to reach the mean, compared to only 1.5 in China and 1.4 in India.

Finally, the measure $P_2(y;z) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{z}{y_i}\right)$ associated with $I_2(y)$ also has desirable properties and lends itself to intuitive interpretation. Sterck (2024) provides a conceptual justification and an axiomatization of $P_2(y;z)$ in the context of poverty measurement, characterizing it as an inclusive measure of poverty, which can be interpreted as the average number of days needed to get z. In the context of Shared Prosperity measurement, Kraay et al. (2024) describe $P_2(y;z)$ as the prosperity gap, which is the average factor by which incomes must be multiplied to reach z. The measure $P_2(y;z)$ has excellent properties, being distribution sensitive and satisfying key axioms in the measurement of welfare and poverty (except focus). Equation (16) offers a simple but powerful framework linking inequality, average income, and average poverty, in line with Bourguignon (2003, 2004)'s Poverty-Growth-Inequality Triangle.[13] Reflecting this relevance, the World Bank has adopted $P_2(y;z)$ as its core metrics for monitoring progress toward its goal to promote Shared Prosperity.

4 Application

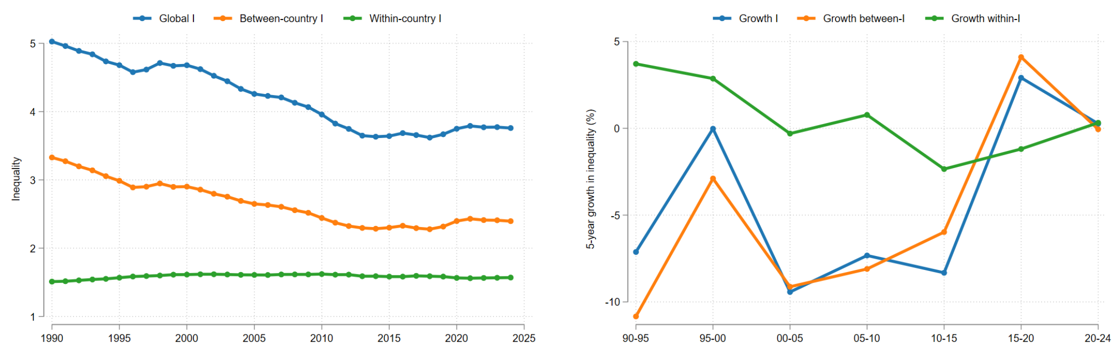
We illustrate the usefulness of our framework through two applications: one examining global inequality trends and another focusing on inequality within the United States. Existing inequality measures typically offer either strong theoretical properties or some intuitive appeal, but never both. Measures in the Generalized Entropy class, for instance, are widely used for their decomposability but are often seen as difficult to interpret. In contrast, measures such as the Gini coefficient or quantile ratios (e.g., Palma) are more easily understood but lack key properties, especially decomposability and transfer

[13] To be sure, similar relationships have been formulated for other welfare indicators and poverty measures (Atkinson 1970; Sen 1976). Yet, the distinct strength of the framework formed by $I_2(y)$, $P_2(y;z)$, and μ is that all measures have excellent properties and an intuitive interpretation, forming a coherent framework that links changes in inequality, prosperity, and poverty.

sensitivity. In both applications, we focus on the Average Inequality Ratio, the central measure in our class. Our goal is to demonstrate that this measure offers both clean decompositions and an intuitive interpretation, making it a useful tool for both researchers and policymakers.

4.1 Global Inequality

We illustrate the decomposition properties of average inequality—$I_2(y)$—using data from the global interpersonal income distribution available through the World Bank's Poverty and Inequality Platform (PIP), as of September 19, 2024. PIP is a comprehensive database comprising approximately 2,500 household surveys from 168 countries, covering over 97 percent of the world's population. It provides global income distributions from 1990 to 2024, with each country-year represented by 1,000 income bins (Mahler et al. 2022).[14] Income is expressed in constant 2017 Purchasing Power Parity (PPP) dollars, per person per day. As Kraay et al. (2024), we bottom-code all values at $0.25 per day to prevent extremely low reported incomes from disproportionately influencing the estimates.



(a) Inequality trends

(b) 5-year growth rates

Figure 2: Multiplicative decomposition: evolution of between- and within-country inequality over time

Data source: PIP data (World Bank). Income is measured at 2017 $PPP per person per day and the distribution is bottom coded at $0.25 (see Kraay et al. 2024 for a discussion).

[14]Due to variation in data sources, PIP includes surveys that use consumption as the primary measure of household welfare (covering roughly three-quarters of the global population), alongside those that rely on income. Following the World Bank's practice in constructing global poverty estimates, we do not distinguish between income- and consumption-based measures when aggregating cross-country distributions.

Figure 2 illustrates the decomposition of average inequality into between- and within-country components, as defined in equation 2. Panel (a) presents the evolution of average inequality over time, while Panel (b) applies a logarithmic transformation to express the multiplicative decomposition as an additive decomposition of inequality growth rates, exploiting the link between logarithms and growth. The results show that global inequality declined steadily between 1990 and 2014, before experiencing a modest increase, particularly between 2018 and 2021. This evidence of a reversing trend is consistent with recent observations by Deaton (2021), Kanbur et al. (2022), and Milanovic (2024). In 2024, Average Inequality was about 5, meaning that the expected income ratio of two randomly selected individuals was 5. This figure had reached a low of about 3.6 in 2018, before rising again to 3.8 in 2024. Therefore, the expected income ratio between two randomly selected individuals globally was around 3.8 in 2024.

Throughout the 1990-2024 period, changes in global inequality were driven primarily by shifts in between-country inequality, while within-country inequality remained relatively stable. This result is consistent with the literature on global inequality decomposition (Lakner and Milanovic 2016; Kanbur 2019; Milanovic 2024) and on the convergence of low- and middle-income countries (Patel et al. 2021; Kremer et al. 2022).

Figure 3 offers an alternative visualization of the same data, showing how the between- and within-country components interact to jointly determine total inequality. It shows that the long-term decline in global inequality from 1990 to 2024 was largely the result of falling between-country inequality. By contrast, within-country inequality rose slightly over the same period, but its contribution to overall inequality remained limited.

Table 1 presents the components of the additive decomposition of average inequality by country, focusing on the 25 countries that contribute most to global inequality. The final column shows each country's overall contribution. We identify three distinct groups among these top contributors, based on the primary source of their contribution. First, large middle-income countries such as India, China, Nigeria, Pakistan, and Indonesia appear on the list primarily due to their large populations and relatively low average incomes, which result in significant contributions to between-country inequality. This

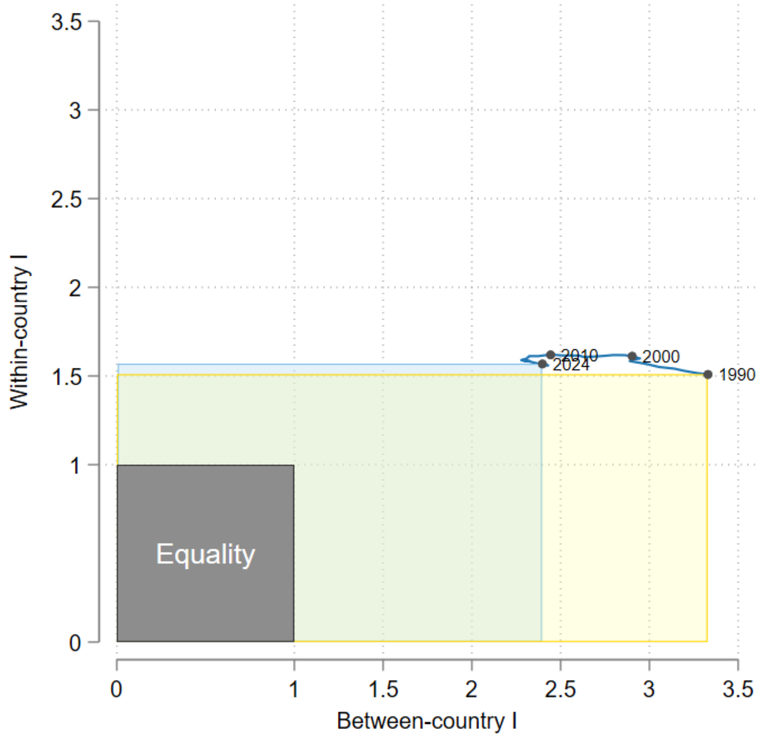


Figure 3: Multiplicative decomposition: interaction of between- and within-country components over time

Data source: PIP data (World Bank). Income is measured at 2017 $PPP per person per day and the distribution is bottom coded at $0.25 (see Kraay et al. 2024 for a discussion).

is reflected in the high weight $\omega_g(2)$ in the decomposition. Second, several low-income countries—such as the Democratic Republic of Congo, Mozambique, Madagascar, Sudan, and Yemen—contribute heavily because of very low average incomes, despite modest population sizes and within-country inequality levels. Third, countries like Brazil, the United States, and South Africa are included due to the high levels of inequality within these countries.

Figure 4 provides a visual representation of these results, also showing trends in the contributions of the most populous countries over time. A country's contribution depends on two key factors: average inequality within the country, shown on the vertical axis, and its weight, shown on the horizontal axis. This weight is proportional to population size and inversely proportional to average income.

Countries such as India and China appear in the bottom-right quadrant of the figure: their high weight reflects large populations relative to average income. In contrast,

Table 1: Inequality decompositions: country contributions and weights in 2024 (top 25)

Country	Population (million)	Average income \$/day	Average poverty day/2.15\$	Prosperity gap	Average inequality $I_g(2)$	Multiplicative decomposition $\nu_g(2)$	Additive decomposition $\omega_g(2)$	Country Contribution $\omega_g(2) \times I_g(2)$
1. India	1442	5.5	0.55	6.4	1.4	0.26	0.62	0.87
2. China	1412	16.6	0.19	2.2	1.5	0.08	0.20	0.29
3. Congo, Dem. Rep.	106	1.9	2.02	23.5	1.8	0.05	0.13	0.23
4. Nigeria	229	3.6	0.88	10.2	1.5	0.06	0.15	0.22
5. Pakistan	245	5.2	0.53	6.2	1.3	0.05	0.11	0.14
6. Indonesia	280	7.9	0.39	4.6	1.5	0.03	0.08	0.12
7. Ethiopia	130	4.8	0.66	7.6	1.5	0.03	0.06	0.09
8. Bangladesh	175	6.4	0.47	5.4	1.4	0.03	0.06	0.09
9. Mozambique	35	2.2	2.06	24	2.1	0.02	0.04	0.08
10. Madagascar	31	1.6	2.28	26.5	1.7	0.02	0.04	0.08
11. Tanzania	69	3.6	0.97	11.3	1.6	0.02	0.05	0.07
12. Brazil	218	22.1	0.27	3.1	2.7	0.01	0.02	0.06
13. Philippines	119	7.3	0.45	5.2	1.5	0.02	0.04	0.06
14. Sudan	49	3.1	1	11.7	1.4	0.02	0.04	0.05
15. Kenya	56	3.8	0.87	10.1	1.6	0.01	0.03	0.05
16. Uganda	50	3.7	0.98	11.4	1.7	0.01	0.03	0.05
17. Egypt, Arab Rep.	114	7.5	0.39	4.5	1.3	0.02	0.04	0.05
18. Yemen, Rep.	35	2.6	1.23	14.3	1.5	0.01	0.03	0.05
19. United States	337	82.7	0.13	1.5	4.9	0.00	0.01	0.05
20. Zambia	21	2.6	1.97	22.9	2.4	0.01	0.02	0.05
21. Angola	38	5.4	1.06	12.4	2.6	0.01	0.02	0.04
22. South Africa	61	12	0.65	7.6	3.6	0.00	0.01	0.04
23. Malawi	21	2	1.66	19.3	1.6	0.01	0.03	0.04
24. Mexico	129	17.3	0.23	2.6	1.8	0.01	0.02	0.03
25. Niger	28	2.9	1.02	11.9	1.4	0.01	0.02	0.03

Notes: Data source: PIP data (World Bank). Income is measured at 2017 \$PPP per person per day and the distribution is bottom coded at \$0.25 (see Kraay et al. 2024 for a discussion). Average poverty and the prosperity gap are estimated following Sterck (2024) and Kraay et al. (2024). Inequality is estimated using equation (17). Weights are estimated using equations (13) and (14) for $\epsilon = 2$.

countries like Brazil and the United States appear in the top-right quadrant, due to high levels of within-country inequality.

The figure also tracks changes over time for countries with more than 200 million inhabitants. China's contribution to global inequality declined markedly between 1990 and 2024. While its population grew by approximately 24% and within-country inequality remained relatively stable, average income increased eightfold.[15] As a result, China's weight in the global inequality decomposition decreased substantially.

By contrast, the weight assigned to the United States increased significantly over the same period. This rise was driven by increasing within-country inequality, which outpaced economic growth. In 1990, the average income ratio between two randomly selected US individuals was 2.3; by 2024, this ratio had more than doubled to 4.8. In comparison, average income in the US increased by 40% over the same period.

[15]Following equation (16), average poverty fell dramatically over the same period—from 1.5 days to earn \$2.15 in 1990 to just 0.2 days in 2024, an 88% reduction.

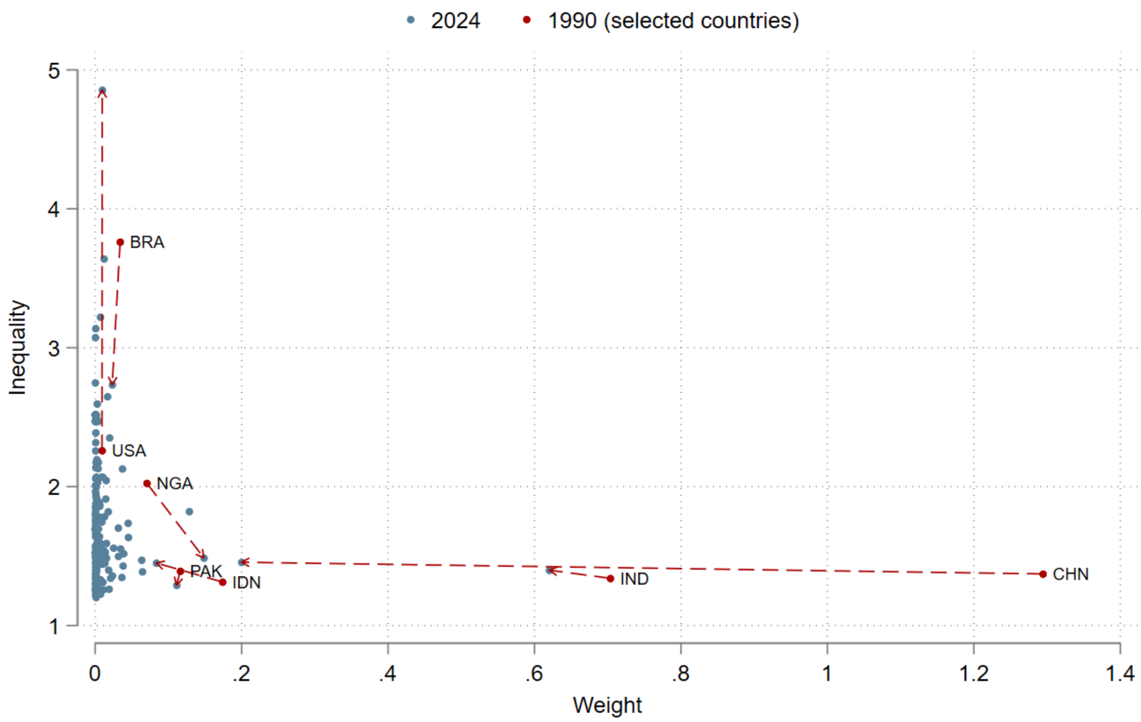


Figure 4: Additive decomposition: country weight vs. within-country inequality in 2024, with evolution over time for countries with more than 200 million inhabitants

Data source: PIP data (World Bank). Income is measured in 2017 \$PPP and the distribution is bottom coded at \$0.25 per person per day (see Kraay et al. 2024 for a discussion). Inequality is estimated using equation (17). Weights are estimated using equation (14) for $\epsilon = 2$.

4.2 Inequality in the US

We further analyze income inequality in the United States, relying on data from IRS Tax Form 1040 on Adjusted Gross Income (AGI). The data was compiled by Rinz and Voorheis (2023) for their study of income convergence across US states. For each state and year from 1998 to 2019, the dataset provides average AGI by percentile. We make two adjustments to the data: (1) we express all incomes in constant 2017 dollars using the CPI-U index; and (2) we linearly interpolate 68 missing data points—representing just 0.06% of the sample—based on adjacent percentiles.

Figure 5 presents the multiplicative decomposition of average income inequality into between- and within-state components. Panel (a) shows the evolution of US inequality over time, while Panel (b) applies a logarithmic transformation to convert the multiplicative decomposition into an additive decomposition of inequality growth rates. The results reveal a general rise in US income inequality between 1998 and 2019, with a temporary

(a) Inequality trends

(b) 5-year growth rates

Figure 5: Multiplicative decomposition: evolution of between- and within-state inequality over time (1998—2019)

Data source: Adjusted gross income (AGI) from IRS Tax Form 1040, as provided by Rinz and Voorheis (2023).

spike during the 2008 financial crisis.

In contrast to global inequality, income inequality in the United States is driven primarily by disparities within states. Take California—the most populous state—as an example: in 2019, daily incomes ranged from \$3.4 at the bottom percentile and \$36 at the 10th percentile to \$572 at the 90th percentile and \$7,379 at the top percentile. By comparison, inequality between states is minimal. In 2019, average daily incomes ranged from \$170 in Mississippi to \$393 in Washington, District of Columbia.

Between 1998 and 2019, changes in U.S. income inequality were overwhelmingly driven by shifts in within-state inequality, while between-state inequality remained low and remarkably stable.[16] The persistence of low between-state inequality aligns with the literature on income convergence, which finds that the rapid convergence of per-capita incomes across US states observed before the 1990s has slowed in recent decades (Barro and Sala-i Martin 1992; Ganong and Shoag 2017; Rinz and Voorheis 2023).

Figure 6 presents the components of the additive decomposition of US inequality by state (see Appendix Table A.1 for details). It also highlights trends over time for states with either a high contribution to US inequality (greater than 0.1) or high within-state inequality (greater than 5). A state's contribution is determined by two factors: its average within-state inequality, shown on the vertical axis, and its weight in the national

[16] Appendix Figure A.1 visualizes the interaction between between- and within-state components, further emphasizing that national inequality is largely driven by within-state variation.

decomposition, shown on the horizontal axis. The weight is proportional to the state's population size and inversely proportional to its average income.

States such as California and Texas appear on the right-hand side of the figure, reflecting their substantial weight. These larger weights are primarily driven by their large populations, since average income levels across states are relatively similar. By contrast, states like Alaska, Connecticut, New York, and Washington, D.C. appear on the upper-left quadrant. These states exhibit exceptionally high within-state inequality but have relatively small populations, which reduces their overall weight in the national decomposition.

The figure also tracks changes over time for a subset of large states and states with particularly high inequality. In fact, within-state inequality has increased over time in every state—including those not shown in the figure. Inequality is generally very high in all US states and has been increasing over the past decades. For example, in California, the expected income ratio between two randomly selected individuals rose from 3.4 in 1998 to 4.7 in 2019. The increase was even more pronounced in Alaska, where the same ratio grew from 5.2 to 7.8 over the same period.

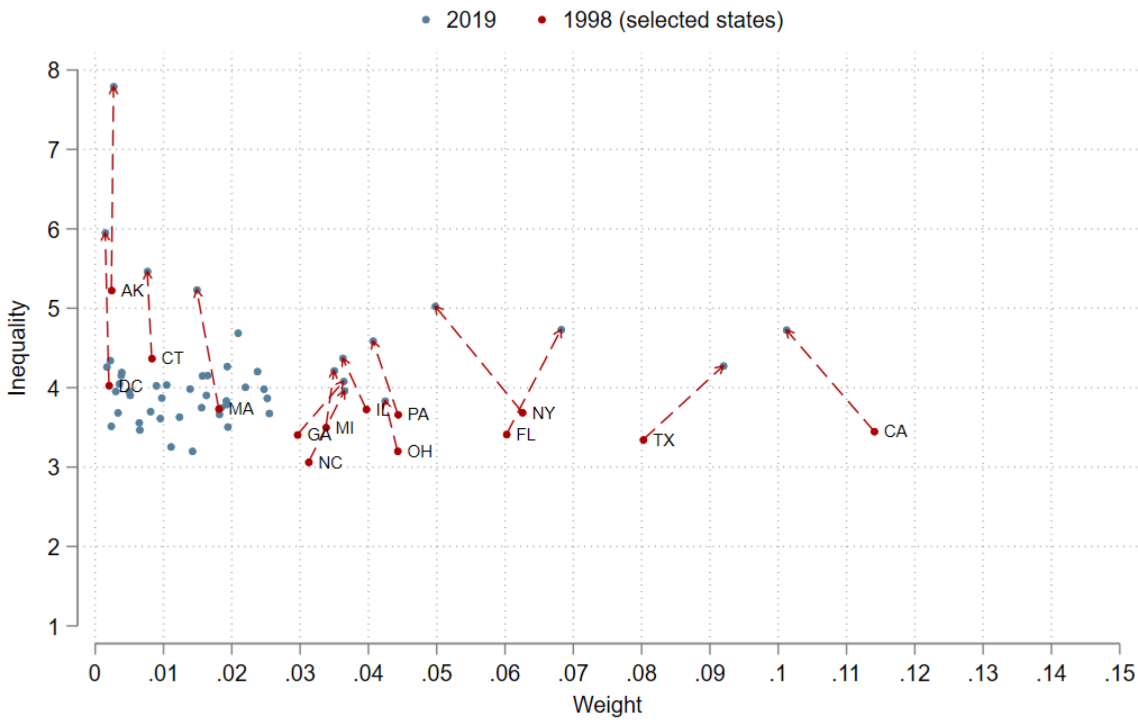


Figure 6: Additive decomposition: state weight vs. within-state inequality in 2019, with evolution over time for selected states (1998—2019)

Data source: Adjusted gross income (AGI) from IRS Tax Form 1040, as provided by Rinz and Voorheis (2023). Inequality is estimated using equation (17). Weights are estimated using equation (14) for $\epsilon = 2$.

5 Conclusion

This paper introduced a new class of inequality measures that expands the analytical toolkit for decomposing income inequality. Building on alternative forms of decomposition—multiplicative within- and between-group decomposition, and additive decomposition by subgroups—we derived a single-parameter family of inequality indices that satisfy both forms under weak assumptions. The weights used in these decompositions offer a transparent interpretation: they sum to one in the multiplicative case and vary systematically with subgroup population and income levels. For $\epsilon > 1$, the decompositions give greater emphasis to inequality in large and poor populations.

Among this class, the measure with an inequality aversion parameter of $\epsilon = 2$ stands out for both its empirical relevance and intuitive appeal. It is the inequality measure corresponding to the Prosperity Gap, a measure was adopted by the World Bank to monitor Shared Prosperity, one if its key objectives. Its interpretation is intuitive. It corresponds to the expected income ratio between two randomly selected individuals. Alternatively, the measure can be understood as the average number of days an individual would need to get the mean income, or as the average factor by which individual incomes must be multiplied to reach the mean.

Taken together, our findings provide a new perspective on inequality measurement and open new avenues for empirical research. The proposed class of measures allows for more interpretable and policy-relevant decompositions of inequality—both in cross-sectional analyses and over time. Its direct connection to average poverty and average income offers a coherent and unified framework for jointly monitoring inequality, poverty, and prosperity, making it well-suited for both academic research and practical policy evaluation.

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Online Appendix

A Proofs

A.1 Proof of Lemma 3

Proof. Take any partition of the population and let subgroup 1 be any subgroup containing two or more individuals. If this subgroup has n_1 individuals with distribution y^1 and mean μ_1, define x^1 to be another distribution over n_1 individuals with the same mean μ_1 such that $I(x^1) \neq I(y^1)$. Normalization ensures that a suitable choice of x^1 can always be made.

The distributions $y = \left(y^1, y^2, ..., y^G\right)$ and $x = \left(x^1, y^2, ..., y^G\right)$ have both the same distribution of means and subgroup population: $\boldsymbol{\mu}, \boldsymbol{n}$. Therefore, applying Additive decomposability in subgroups

$$\begin{aligned} I(y) - I(x) &= \left(\sum_{g=1}^{G} \omega_g(\boldsymbol{\mu}, \boldsymbol{n}) I(y^g)\right) - \left(\sum_{g=1}^{G} \omega_g(\boldsymbol{\mu}, \boldsymbol{n}) I(x^g)\right) \\ &= \omega_1(\boldsymbol{\mu}, \boldsymbol{n}) \left[I\left(y^1\right) - I\left(x^1\right)\right]. \end{aligned} \tag{18}$$

Since $I(y) - I(x)$ is unchanged if we partition $y = \left(y^1, \left(y^2, ..., y^G\right)\right)$ and $x = \left(x^1, \left(y^2, ..., y^G\right)\right)$,

$$I(y) - I(x) = \omega_1(\mu_1, \frac{n\mu - n_1\mu_1}{n - n_1}, n_1, n - n_1) \left[I\left(y^1\right) - I\left(x^1\right)\right]. \tag{19}$$

So in general $\omega_g(\boldsymbol{\mu}, \boldsymbol{n}) = \omega\left(Y_g, Y, n_g, n\right)$ where $Y_g = n_g\mu_g$ and $Y = \mu n$.
Now take the partitions $y = \left(\left(y^1, y^2\right), \left(y^3, ..., y^G\right)\right)$ and $x = \left(\left(x^1, y^2\right), \left(y^3, ..., y^G\right)\right)$. Once again, the different partition does not change $I(y) - I(x)$:

$$I(y) - I(x) = \omega\left(Y_1 + Y_2, Y, n_1 + n_2, n\right) \left[I\left(y^1, y^2\right) - I\left(x^1, y^2\right)\right]. \tag{20}$$

Observe that $I\left(y^1, y^2\right) - I\left(x^1, y^2\right) = \omega(Y_1, Y_1 + Y_2, n_1, n_1 + n_2)\left[I\left(y^1\right) - I\left(x^1\right)\right]$. Therefore,

$$I(y) - I(x) = \omega\left(Y_1 + Y_2, Y, n_1 + n_2, n\right) \left\{\omega(Y_1, Y_1 + Y_2, n_1, n_1 + n_2) \left[I\left(y^1\right) - I\left(x^1\right)\right]\right\} \tag{21}$$

Moreover, we have shown before–equation (19)—that $I(y) - I(x) = \omega(Y_1, Y, n_1, n)\left[I\left(y_1\right) - I\left(x_1\right)\right]$. Combining (19) and (21), and rearranging, we get

$$\omega(Y_1, Y_1 + Y_2, n_1, n_1 + n_2) = \frac{\omega\left(Y_1, Y, n_1, n\right)}{\omega\left(Y_1 + Y_2, Y, n_1 + n_2, n\right)}.$$

Keeping $Y = \tilde{Y}$ and $n = \tilde{n}$ constant, and defining $\theta\left(\mu_g, n_g\right) = \omega\left(n_g\mu_g, \tilde{Y}, n_g, \tilde{n}\right)$, we obtain

$$\omega(Y_1, Y_1+Y_2, n_1, n_1+n_2) = \frac{\omega\left(Y_1, \tilde{Y}, n_1, \tilde{n}\right)}{\omega\left(Y_1+Y_2, \tilde{Y}, n_1+n_2, \tilde{n}\right)}$$

$$= \frac{\theta\left(\mu_1, n_1\right)}{\theta\left(\frac{n_1\mu_1+n_2\mu_2}{n_1+n_2}, n_1+n_2\right)} = \frac{\theta\left(\mu_1, n_1\right)}{\theta\left(\mu, n\right)}.$$

Thus, in general,

$$\omega_g\left(\boldsymbol{\mu}, \boldsymbol{n}\right) = \omega\left(Y_g, Y, n_g, n\right) = \frac{\theta\left(\mu_g, n_g\right)}{\theta\left(\mu, n\right)}.$$

Therefore,

$$I\left(y\right) = \sum_{g=1}^{G} \frac{\theta\left(\mu_g, n_g\right)}{\theta\left(\mu, n\right)} I\left(y^g\right).$$

□

A.2 Proof of Lemma 4

Proof. Let i, j be two individuals and chose any partition in which $y^1 = (y_i, y_j)$. Let $x^1 = (\mu_1, \mu_1)$ where $\mu_1 = (y_i + y_j)/2$. Then, substituting (8) into (19) gives

$$I\left(\left(y_i, y_j\right), y^2, ..., y^G\right) - I\left(\left(\mu_1, \mu_1\right), y^2, ..., y^G\right) = \frac{\theta\left(\mu_1, 2\right)}{\theta\left(\mu, n\right)}\left[I\left(y_i, y_j\right) - I\left(\mu_1, \mu_1\right)\right]$$

$$I\left(\left(y_i, y_j\right), y^2, ..., y^G\right) = \frac{\theta\left(\mu_1, 2\right)}{\theta\left(\mu, n\right)}\left[I\left(y_i, y_j\right) - I\left(\mu_1, \mu_1\right)\right] + I\left(\left(\mu_1, \mu_1\right), y^2, ..., y^G\right)$$

$$I\left(\left(y_i, y_j\right), y^2, ..., y^G\right) = \frac{\theta\left(\mu_1, 2\right)}{\theta\left(\mu, n\right)}\left[I\left(y_i, y_j\right) - 1\right] + I\left(\left(\mu_1, \mu_1\right), y^2, ..., y^G\right).$$

Differentiating with respect to y_i and y_j gives:

$$I_i\left(y\right) = \frac{\partial \frac{\theta(\mu_1,2)}{\theta(\mu,n)}}{\partial y_i} I\left(y_i, y_j\right) + \frac{\theta\left(\mu_1, 2\right)}{\theta\left(\mu, n\right)} \frac{\partial I\left(y_i, y_j\right)}{\partial y_i} - \frac{\partial \frac{\theta(\mu_1,2)}{\theta(\mu,n)}}{\partial y_i} + \frac{\partial I\left(\left(\mu_1, \mu_1\right), y_2 ..., y_G\right)}{\partial \mu_1} \frac{\partial \mu_1}{\partial y_i}$$

$$I_j\left(y\right) = \frac{\partial \frac{\theta(\mu_1,2)}{\theta(\mu,n)}}{\partial y_j} I\left(y_i, y_j\right) + \frac{\theta\left(\mu_1, 2\right)}{\theta\left(\mu, n\right)} \frac{\partial I\left(y_i, y_j\right)}{\partial y_j} - \frac{\partial \frac{\theta(\mu_1,2)}{\theta(\mu,n)}}{\partial y_j} + \frac{\partial I\left(\left(\mu_1, \mu_1\right), y_2 ..., y_G\right)}{\partial \mu_1} \frac{\partial \mu_1}{\partial y_j}.$$

We can now compute $I_i\left(y\right) - I_j\left(y\right)$. Notice that $\frac{\partial \mu_1}{\partial y_i} = \frac{\partial \mu_1}{\partial y_j} = \frac{1}{2}$. Moreover, $\frac{\partial \frac{\theta(\mu_1,2)}{\theta(\mu,n)}}{\partial y_i} =$

$\frac{\partial \frac{\theta(\mu_1,2)}{\theta(\mu,n)}}{\partial y_j}$. Indeed, $\frac{\partial \frac{\theta(\mu_1,2)}{\theta(\mu,n)}}{\partial y_i}$ depends on $\frac{\partial \mu_1}{\partial y_i}$ and $\frac{\partial \mu}{\partial y_i}$, which are the same for y_i and y_j. Thus,

$$I_i(y) - I_j(y) = \frac{\theta(\mu_1, 2)}{\theta(\mu, n)} \frac{\partial I(y_i, y_j)}{\partial y_i} - \frac{\theta(\mu_1, 2)}{\theta(\mu, n)} \frac{\partial I(y_i, y_j)}{\partial y_i}$$

$$\theta(\mu, n)[I_i(y) - I_j(y)] = \theta(\mu_1, 2)\left(\frac{\partial I(y_i, y_j)}{\partial y_i} - \frac{\partial I(y_i, y_j)}{\partial y_j}\right).$$

Hence,

$$\begin{aligned} \theta(\mu, n)[I_i(y) - I_j(y)] = & \quad \theta\left(\tfrac{y_i+y_j}{2}, 2\right)(I_1(y_i, y_j) - I_2(y_i, y_j)) \\ = & \quad f(y_i, y_j) \end{aligned} \tag{22}$$

for some function f, and

$$\begin{aligned} f(y_i, y_j) + f(y_j, y_k) = & \quad \theta(\mu, n)[I_i(y) - I_j(y)] + \theta(\mu, n)[I_j(y) - I_k(y)] \\ = & \quad \theta(\mu, n)[I_i(y) - I_k(y)] \\ = & \quad f(y_i, y_k) \end{aligned}$$

for all y_i, y_j, y_k.
Now,

$$f(y_i, y_j) + f(y_j, y_k) = f(y_i, y_k) \iff f(y_i, y_j) = f(y_i, y_k) - f(y_j, y_k)$$

and if we define $\phi'(a) = f(a, 0)$, we can rewrite this as

$$f(y_i, y_j) = f(y_i, 0) - f(y_j, 0) = \phi'(y_i) - \phi'(y_j). \tag{23}$$

Notice that here we have defined a function $\phi(t)$, $t \in \mathbb{R}_+$, whose derivative corresponds to $f(t, 0)$. Combining (22) and (23),

$$\theta(\mu, n)[I_i(y) - I_j(y)] = \phi'(y_i) - \phi'(y_j)$$

$$\theta(\mu, n) I_i(y) - \phi'(y_i) = \theta(\mu, n) I_j(y) - \phi'(y_j).$$

Observe that on each side we have two derivatives of two different functions (I and ϕ) with respect to the same variable: y_i on the left and y_j on the right. We can write each side as the derivative of a function $g(y)$ with respect to the considered individual incomes $y_i$and y_j in y:

$$\underbrace{\theta(\mu, n) I_i(y) - \phi'(y_i)}_{g_i(y)} = \underbrace{\theta(\mu, n) I_j(y) - \phi'(y_j)}_{g_j(y)}.$$

We thus have that $g_i(y) = g_j(y)$ for all i, j, [17] and

$$g(y) = \theta(\mu, n) I(y) - \sum_{i=1}^{n} \phi(y_i). \tag{24}$$

Because all derivatives are the same ($g_i(y) = g_j(y)$) the function g should only depend on the total income in y, which is a function of μ and n. Thus, we can set $g(y) = \beta(\mu, n)$ for some function β. However,

$$\beta(\mu, n) = g(\mathbf{1}_n \mu) = \theta(\mu, n) \underbrace{I(\mathbf{1}_n \mu)}_{=1} - \sum_{i=1}^{n} \phi(\mu) = \theta(\mu, n) - \sum_{i=1}^{n} \phi(\mu) \tag{25}$$

Hence, combining (24) and (25)

$$\theta(\mu, n) I(y) - \sum_{i=1}^{n} \phi(y_i) = \theta(\mu, n) - \sum_{i=1}^{n} \phi(\mu).$$

$$\theta(\mu, n) I(y) = \theta(\mu, n) + \sum_{i=1}^{n} [\phi(y_i) - \phi(\mu)]$$

$$I(y) = 1 + \frac{1}{\theta(\mu, n)} \sum_{i=1}^{n} [\phi(y_i) - \phi(\mu)].$$

That θ_μ and ϕ' are continuous follows from the differentiability of I (imposed by Continuity). Without loss of generality we may take $\theta(\mu, n) > 0$. The Transfer Principle implies that ϕ is strictly convex.

This completes the necessity part of the proof. Sufficiency is straightforward and left to the reader. □

A.3 Proof of Lemma 5

Proof. From (9) we obtain

$$I(y) = 1 + \frac{1}{\theta(\mu, n)} \sum_{i=1}^{n} [\phi(y_i) - \phi(\mu)]$$

$$I\left(\underbrace{y, \ldots, y}_{\times m}\right) = 1 + \frac{m}{\theta(\mu, mn)} \sum_{i=1}^{n} [\phi(y_i) - \phi(\mu)].$$

Replication invariance holds if and only if

$$1 + \frac{1}{\theta(\mu, n)} \sum_{i=1}^{n} [\phi(y_i) - \phi(\mu)] = 1 + \frac{m}{\theta(\mu, mn)} \sum_{i=1}^{n} [\phi(y_i) - \phi(\mu)]$$

[17] Thus, $g(y)$ is a linear function whose total differentiation corresponds to $\theta(\mu, n) \sum_{i=1}^{n} I_i(y) - \sum_{i=1}^{n} \phi'(y_i)$.

$$\theta\left(\mu, mn\right) = m\theta\left(\mu, n\right).$$

for all $n \geq 2$. So we can write $2\theta\left(\mu, n\right) = \theta\left(\mu, 2n\right) = n\theta\left(\mu, 2\right)$, which gives

$$\theta\left(\mu, n\right) = n\frac{\theta\left(\mu, 2\right)}{2} = n\alpha\left(\mu\right)$$

where α is positive and continuously differentiable. $\square$

B Additional Figures and Tables

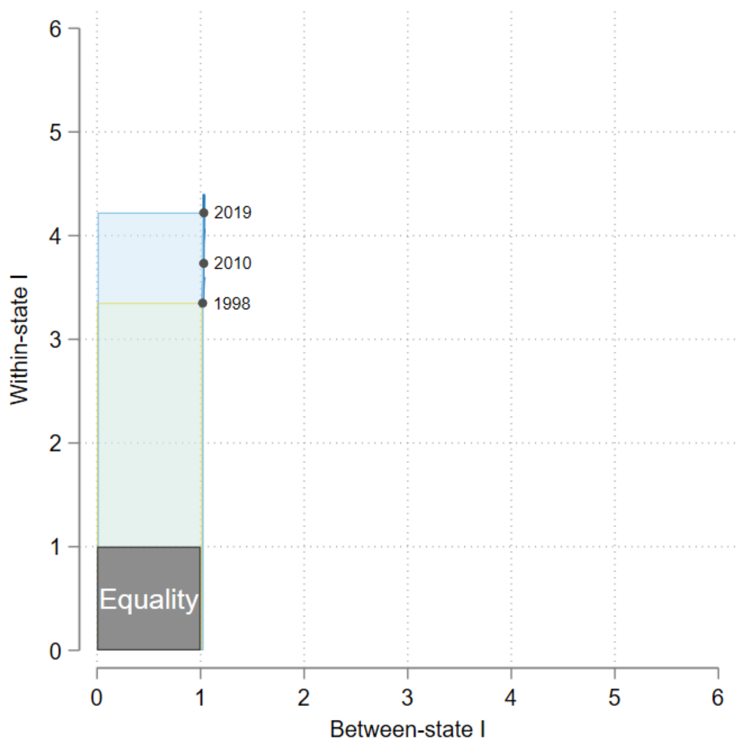

Figure A.1: Multiplicative decomposition: interaction of between- and within-state components over time

Data source: Adjusted gross income (AGI) from IRS Tax Form 1040, as provided by Rinz and Voorheis (2023).

Table A.1: Inequality decompositions: US States contributions and weights in 2019

State	Population (million)	Average income \$/day	Average poverty day/2.15\$	Prosperity gap	Average inequality $I_g(2)$	Multiplicative decomposition $\nu_g(2)$	Additive decomposition $\omega_g(2)$	State Contribution $\omega_g(2) \times I_g(2)$
Alabama	5	206	0.037	0.43	3.5	0.02	0.02	0.07
Alaska	1	222	0.075	0.88	7.8	0.00	0.00	0.02
Arizona	7	232	0.034	0.4	3.7	0.02	0.03	0.09
Arkansas	3	199	0.039	0.46	3.6	0.01	0.01	0.04
California	40	318	0.032	0.37	4.7	0.10	0.10	0.48
Colorado	6	299	0.03	0.35	4.2	0.02	0.02	0.07
Connecticut	4	381	0.031	0.36	5.5	0.01	0.01	0.04
Delaware	1	240	0.033	0.38	3.7	0.00	0.00	0.01
District of Columbia	1	393	0.033	0.38	6	0.00	0.00	0.01
Florida	21	256	0.04	0.46	4.7	0.07	0.07	0.32
Georgia	11	238	0.037	0.43	4.1	0.04	0.04	0.15
Hawaii	1	234	0.036	0.42	4	0.00	0.00	0.02
Idaho	2	227	0.034	0.39	3.6	0.01	0.01	0.02
Illinois	13	285	0.033	0.38	4.4	0.04	0.04	0.16
Indiana	7	218	0.038	0.44	3.9	0.02	0.03	0.10
Iowa	3	232	0.03	0.35	3.3	0.01	0.01	0.04
Kansas	3	244	0.034	0.4	3.9	0.01	0.01	0.04
Kentucky	4	200	0.039	0.46	3.7	0.02	0.02	0.07
Louisiana	5	206	0.039	0.45	3.7	0.02	0.02	0.07
Maine	1	215	0.039	0.45	3.9	0.00	0.01	0.02
Maryland	6	300	0.03	0.35	4.2	0.02	0.02	0.07
Massachusetts	7	377	0.03	0.35	5.2	0.01	0.01	0.08
Michigan	10	232	0.039	0.45	4.2	0.03	0.03	0.15
Minnesota	6	282	0.03	0.35	3.9	0.02	0.02	0.06
Mississippi	3	170	0.04	0.47	3.2	0.01	0.01	0.05
Missouri	6	227	0.038	0.44	4	0.02	0.02	0.09
Montana	1	226	0.04	0.46	4.2	0.00	0.00	0.02
Nebraska	2	242	0.031	0.36	3.5	0.01	0.01	0.02
Nevada	3	240	0.036	0.42	4	0.01	0.01	0.04
New Hampshire	1	293	0.031	0.36	4.2	0.00	0.00	0.02
New Jersey	9	346	0.029	0.34	4.7	0.02	0.02	0.10
New Mexico	2	191	0.045	0.53	4	0.01	0.01	0.04
New York	19	318	0.034	0.4	5	0.05	0.05	0.25
North Carolina	10	234	0.036	0.42	4	0.04	0.04	0.14
North Dakota	1	268	0.028	0.33	3.5	0.00	0.00	0.01
Ohio	12	224	0.037	0.43	3.8	0.04	0.04	0.16
Oklahoma	4	207	0.039	0.45	3.8	0.02	0.02	0.06
Oregon	4	248	0.035	0.4	4	0.01	0.01	0.06
Pennsylvania	13	256	0.039	0.45	4.6	0.04	0.04	0.19
Rhode Island	1	246	0.035	0.41	4.1	0.00	0.00	0.01
South Carolina	5	218	0.037	0.43	3.8	0.02	0.02	0.07
South Dakota	1	242	0.035	0.41	4	0.00	0.00	0.01
Tennessee	7	225	0.038	0.44	4	0.02	0.02	0.10
Texas	29	256	0.036	0.42	4.3	0.09	0.09	0.39
Utah	3	275	0.028	0.33	3.6	0.01	0.01	0.03
Vermont	1	234	0.04	0.46	4.3	0.00	0.00	0.01
Virginia	9	293	0.031	0.36	4.2	0.02	0.02	0.10
Washington	8	321	0.029	0.33	4.3	0.02	0.02	0.08
West Virginia	2	181	0.044	0.51	3.7	0.01	0.01	0.03
Wisconsin	6	247	0.033	0.39	3.8	0.02	0.02	0.07
Wyoming	1	281	0.033	0.38	4.3	0.00	0.00	0.01

Notes: Data source: Adjusted gross income (AGI) from IRS Tax Form 1040, as provided by Rinz and Voorheis (2023). Income is expressed in 2017 US\$. Average poverty and the prosperity gap are estimated following Sterck (2024) and Kraay et al. (2024). Inequality is estimated using equation (17). Weights are estimated using equations (13) and (14) for $\epsilon = 2$.