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Nonparametric Mixed Frequency Monitoring Macro-at-Risk

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Executive Summary

We compare homoskedastic and heteroskedastic mixed frequency (MF) vector autoregression and Bayesian additive regression tree (BART) models to assess their relative performance in predicting tail risk. MF-BART is a nonlinear state space model, and we discuss linear approximation approaches to devise computationally efficient estimation algorithms. The models are applied in an out-of-sample backcasting, nowcasting and forecasting exercise for a set of quarterly and monthly macroeconomic variables in Italy. The proposed econometric refinements yield improvements in predictive accuracy.

Introduction

Quantifying macroeconomic risk has received growing attention from academics and policymakers, following the influential work of Adrian et al. (2019). Their "growth-at-risk" framework has been applied to other indicators such as inflation or debt (e.g., Adams et al., 2021; Lopez-Salido and Loria, 2024; Furceri et al., 2025). Most related papers use quantile regressions with quarterly data. Some recent empirical evidence suggests that multivariate models, like vector autoregressions (VARs) with heteroskedastic errors, or nonparametric versions of such models, can significantly improve predictive accuracy. Further, using higher-frequency information to predict lower-frequency target variables has proven useful both in single and multiple equation models. Motivated by these aspects, in Marcellino and Pfarrhofer (2025), we consider parametric and nonparametric versions of a multivariate mixed frequency (MF) model. As a baseline, we use the nonparametric MF Bayesian Additive Regression Tree (MF-BART) model of Huber et al. (2023), subject to several econometric refinements—indeed, we propose an alternative approximation step for linearizing the underlying nonlinear state space model. Further, we provide a computationally efficient, precision-based, estimation algorithm adapted from Chan et al. (2023), which is scalable to high dimensions– a crucial feature for practitioners and professional forecasters.

Our empirical work complements Boeck et al. (2025), who conduct a forecasting horserace for predicting tail risk in Italy with a similar dataset. By contrast, our focus is on short-horizon forecasts, nowcasts, and backcasts. The latter two types of predictions are necessary due to the significant publication lags of key variables. And instead of assessing predictions of models estimated separately for quarterly and monthly data, we rely on a joint MF framework in an out-of-sample evaluation exercise. We pick Italy due to its high level of public debt, where risk monitoring is especially important—but the methods are generally applicable. As quarterly targets, we focus on the deficit- and debt-to-GDP ratios, and real GDP. Further, we assess monthly predictions for unemployment, industrial production and inflation. We find MF-VARs equipped with a global-local shrinkage prior and heteroskedastic errors to perform well in predicting upside and downside risk. MF-BART typically exhibits a comparable performance to its linear competitor, although there are noteworthy gains in predictive accuracy for some variables at several horizons. Compared with the original MF-BART model implementation of Huber et al. (2023) our version offers additional improvements in predictions.

Econometric Framework

We consider a set of variables on a quarterly frequency that we model alongside monthly variables in a joint framework. The key challenge in this context is that in terms of the monthly frequency, the quarterly observations are only measured each third month, and there are missing values in between. We follow the literature on MF-VARs and assume that the quarterly values are linked to an unobserved process via a so-called intertemporal restriction. Combining this intertemporal restriction with an equation that governs the dynamic evolution of the unobserved states, we obtain a state space model. When assuming that the corresponding state equation is linear, standard estimation algorithms can be used. By contrast, when assuming that nonlinearities are present in the dynamic process of the states, this requires a more intricate estimation procedure. Indeed, the latter is what Huber et al. (2023) propose, using a nonparametric conditional mean function estimated with BART.

Our framework builds on this earlier MF-BART model, which relies on a linear approximation of the nonparametric state equation enable a computationally feasible estimation algorithm. The first novelty is that we discuss an alternative approximation procedure which we find to work very well empirically. In the original MF-BART, the linearization approach was operationalized by using a pseudo inverse of the predictor variables to recover linear dynamic VAR coefficients to be used for sampling the latent states, inspired by the earlier works in the machine learning literature. Tree-based implementations such as BART, however, might overfit the data in-sample, and this procedure potentially results in very noisy estimates (comparable to those of an unrestricted VAR) of the coefficients required for filtering. To circumvent this issue, we instead introduce a stochastic approximation error that we implement with a set of auxiliary Bayesian linear regressions subject to regularization with a shrinkage prior.

Another methodological innovation of our proposed framework is to replace the standard filtering/smoothing algorithm that is typically used for estimating such state space models with a so-called precision sampler. Here, we follow Chan et al. (2023) and obtain the moments of several required high-dimensional probability distributions as functions of sparse and banded matrices, which allows for a computationally efficient estimation algorithm. In addition, we equip the model of Huber et al. (2023) with a component that is capable of inferring outliers, which has proven useful in light of the macroeconomic fluctuations observed during the Covid-19 pandemic. This approach can be used to obtain a draw of the unobserved states quickly. Conditional on these states, we may update all remaining model parameters in an otherwise straightforward Markov chain Monte Carlo (MCMC) algorithm (for these parameters, we rely on the established methods in Chipman et al., 2010; Huber et al., 2023).

Monitoring Tail Risk in Italy

We conduct an out-of-sample (OOS) exercise to evaluate the predictive accuracy of a set of popular model specifications that are nested in our framework. These competitors are: (1) Bayesian vector autoregression (BVAR); (2) BART when estimating the conditional mean function using BART (Chipman et al., 2010); these are differentiated with respect to how the linearized version is obtained (see the discussion in "Econometric Framework): Projection (proj), or Horseshoe (hs), when relying on an auxiliary prior specification. Further, we compute and evaluate predictions in two ways: either using the (1) unprocessed (raw) output of the linearized sampling step, or (2) fitting values obtained from inserting the linearly approximated states into the nonlinear function and adding random shocks (nonlinear, nl). Both the BVAR and BART versions are estimated with six lags and either homoskedastic errors (hom), or outlier (o) component.

Dataset and out-of-sample evaluation scheme. Our dataset comprises quarterly and monthly variables from Italy, ranging from January 2001 until June 2024. Specifically, our dataset is patterned after Boeck et al. (2025): Deficit-to-GDP (Deficit) and Debt-to-GDP (Debt) ratio, and real GDP growth (RGDP) as quarterly target variables; Unemployment (in differences), industrial production (IP, annualized log-differences), and inflation (HICP, annualized log-differences) as monthly targets. Further monthly predictors are Italian long-term interest rates (10-year benchmark), the spread between Italian/German long-term government bond yields, economic sentiment indicator, euro area short-term (3-month maturity) rates, and the USD/EUR exchange rate. Inspired by the recent work of Furceri et al. (2025) who focus on predicting "debt-at-risk," we add several timely indicators of economic/policy uncertainty and financial stress: a geopolitical risk indicator (GPR), composite indicator of systemic stress (CISS) for Italy, and European policy uncertainty (EPU).

The initial training sample uses data from the beginning of the sample until January 2010. Because no history of real-time vintages is available, we truncate the final vintage, such that it respects the release calendar (e.g., in the first month of any quarter, the numbers for the previous two quarters and the current quarter of the debt-ratio have not been released yet; in the second month, only the previous and current quarter are missing). This is reflected in our results, which, on the quarterly frequency, indicate the backcasts as $h = -2$ and $h = -1$, the nowcast (present) as $h = 0$ and the forecasts (future) as $h > 1$. Note that in terms of the respective information sets, we assume to compute the predictions at the final day of each month. Our competing models are re-estimated on a monthly basis, adding the most recent available observation for each of the quarterly or monthly indicators. That is, for the quarterly variables we have at least three predictions for the same target quarter (e.g., the $h = 0$ horizon has the 1st, 2nd and 3rd month per quarter).

Forecasting Results. Tables 1 and 2 show quantile-weighted continuous ranked probability scores (CRPSs) as tail forecast metrics. These scoring rules emphasize specific parts of the predictive distribution: downside (left tail, CRPS-L) and upside (right tail, CRPS-R) risk. The rows of the benchmark, BVAR-hom are raw CRPSs (grey shades), all other entries are ratios to this benchmark (blue shading: improvements, red shading vice versa; bold values indicate the best model per variable and horizon). Major columns are horizons (quarters), minor columns indicate months during the quarter. The $h = -2$ column contains only the 1st month indicator, because only "Debt" and "Deficit" are missing (as does the previous quarters' RGDP for the 3rd month of the 1-step backcast). The predictive losses for monthly forecasts are shown in Table 3, where horizons are on a monthly frequency.

The BVAR equipped with heteroskedastic errors performs consistently well across variables and horizons; indeed, this model has been identified as very accurate in earlier related literature. Improvements relative to the linear homoskedastic benchmark are sizable in most cases, for both downside and upside risk. BART(hs)-o often exhibits comparable losses to its linear heteroskedastic competitor. Although there are noteworthy gains in predictive accuracy for some variables and horizons, it must be acknowledged that especially for backcasts and nowcasts of debt, and selected forecasts of the deficit ratio, the metrics indicate a somewhat weaker performance than the linear model. For nowcasts of the monthly variables, the BART-variants are accurate, and best overall in a large number of cases. The performance for forecasts is on par with BVAR-o for unemployment and industrial production, while larger gains arise for inflation, consistent with previous empirical findings.

Variable / Model		Horizon (quarter / month–per–quarter): −2	−1			0			1			2		
		1st	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
Deficit	BVAR–o	**0.78**	1.05	**0.70**	**0.76**	**0.97**	**0.91**	**0.93**	1.09	1.19	**0.91**	1.05	1.07	**0.94**
Deficit	BVAR–hom	0.28	**0.31**	0.25	0.26	0.36	0.29	0.32	**0.37**	**0.28**	0.36	**0.43**	**0.37**	0.42
Deficit	BART(proj)–o [raw]	0.98	1.07	1.01	1.01	1.10	1.10	0.99	1.20	1.36	1.04	1.09	1.15	1.01
Deficit	BART(proj)–o [nl]	0.90	1.03	0.97	0.93	1.09	1.08	0.96	1.19	1.36	1.03	1.09	1.15	1.01
Deficit	BART(proj)–hom [raw]	0.98	1.09	1.00	1.10	1.09	1.08	1.12	1.15	1.28	1.12	1.03	1.05	1.03
Deficit	BART(proj)–hom [nl]	0.95	1.05	0.96	1.05	1.08	1.05	1.09	1.14	1.27	1.11	1.03	1.05	1.03
Deficit	BART(hs)–o [raw]	0.87	1.07	0.89	0.87	1.13	1.07	0.99	1.23	1.34	1.04	1.13	1.15	1.01
Deficit	BART(hs)–o [nl]	0.84	1.03	0.88	0.86	1.11	1.02	0.94	1.22	1.33	1.03	1.13	1.15	1.01
Deficit	BART(hs)–hom [raw]	0.89	1.05	0.91	0.90	1.05	0.99	0.97	1.14	1.19	1.00	1.05	1.05	0.97
Deficit	BART(hs)–hom [nl]	0.86	1.01	0.88	0.88	1.04	0.97	0.94	1.13	1.19	0.99	1.05	1.05	0.97
Debt	BVAR–o	0.93	1.04	0.90	1.05	**0.98**	**0.98**	**0.95**	**0.71**	**0.86**	**0.75**	**0.42**	**0.67**	**0.61**
Debt	BVAR–hom	1.25	**1.23**	1.17	1.16	1.52	1.20	1.18	1.94	1.54	1.70	3.67	1.99	2.20
Debt	BART(proj)–o [raw]	1.23	1.26	1.10	1.24	1.19	1.13	1.20	0.97	0.99	0.93	0.53	0.82	0.76
Debt	BART(proj)–o [nl]	1.11	1.22	1.09	1.14	1.15	1.11	1.16	0.96	0.98	0.93	0.53	0.82	0.76
Debt	BART(proj)–hom [raw]	1.28	1.37	1.17	1.36	1.29	1.24	1.41	1.06	1.09	1.05	0.59	0.87	0.82
Debt	BART(proj)–hom [nl]	1.26	1.34	1.16	1.32	1.25	1.21	1.37	1.05	1.08	1.05	0.59	0.87	0.82
Debt	BART(hs)–o [raw]	**0.85**	1.11	**0.85**	**0.88**	1.06	1.08	1.11	0.88	0.94	0.86	0.50	0.80	0.72
Debt	BART(hs)–o [nl]	0.89	1.09	0.89	0.91	1.02	1.05	1.08	0.87	0.93	0.85	0.50	0.80	0.72
Debt	BART(hs)–hom [raw]	0.95	1.29	0.95	0.94	1.20	1.14	1.18	0.99	0.96	0.89	0.55	0.82	0.74
Debt	BART(hs)–hom [nl]	0.97	1.26	0.97	0.95	1.16	1.10	1.14	0.98	0.96	0.88	0.55	0.82	0.74
RGDP	BVAR–o		**0.80**	**0.83**		**0.85**	**0.84**	**0.79**	**0.77**	**0.74**	**0.82**	**0.70**	**0.73**	**0.69**
RGDP	BVAR–hom		0.34	0.33		0.36	0.35	0.34	0.39	0.41	0.39	0.44	0.41	0.45
RGDP	BART(proj)–o [raw]		0.90	1.01		0.89	0.91	0.89	0.82	0.81	0.85	0.74	0.82	0.74
RGDP	BART(proj)–o [nl]		0.89	0.96		0.90	0.93	0.89	0.82	0.81	0.85	0.74	0.82	0.74
RGDP	BART(proj)–hom [raw]		1.03	1.12		0.99	0.98	1.03	0.97	0.96	1.00	0.85	0.98	0.90
RGDP	BART(proj)–hom [nl]		1.02	1.09		0.99	0.98	1.02	0.98	0.96	1.00	0.85	0.98	0.90
RGDP	BART(hs)–o [raw]		0.85	0.88		0.85	0.84	0.85	0.80	0.76	0.82	0.73	0.79	0.73
RGDP	BART(hs)–o [nl]		0.85	0.86		0.86	0.85	0.84	0.80	0.77	0.82	0.73	0.79	0.73
RGDP	BART(hs)–hom [raw]		0.93	0.96		0.94	0.92	0.92	0.89	0.85	0.91	0.81	0.87	0.82
RGDP	BART(hs)–hom [nl]		0.94	0.97		0.95	0.93	0.93	0.89	0.86	0.92	0.81	0.87	0.82

Table 1: CRPS-L (downside-risk) for quarterly target variables relative (ratios) to the benchmark BVAR-hom (raw predictive losses). Major columns refer to the horizon in quarters, minor columns indicate the month during the quarter the prediction was computed.

Variable / Model		Horizon (quarter / month–per–quarter): −2	−1			0			1			2		
		1st	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
Deficit	BVAR–o	0.90	1.02	**0.76**	0.94	**0.97**	**0.99**	0.98	**0.95**	1.06	**0.96**	**0.85**	1.01	**0.93**
Deficit	BVAR–hom	0.17	0.20	0.16	0.16	0.23	0.18	0.20	0.27	**0.20**	0.23	0.32	**0.24**	0.26
Deficit	BART(proj)–o [raw]	0.95	1.03	0.91	0.96	1.04	1.13	1.04	1.00	1.17	1.05	0.89	1.10	1.03
Deficit	BART(proj)–o [nl]	0.91	1.00	0.90	0.92	1.04	1.11	1.01	1.00	1.16	1.05	0.89	1.10	1.03
Deficit	BART(proj)–hom [raw]	0.98	1.06	0.94	1.03	1.09	1.13	1.11	1.03	1.15	1.09	0.88	1.04	1.01
Deficit	BART(proj)–hom [nl]	0.96	1.04	0.92	1.01	1.08	1.10	1.08	1.03	1.14	1.09	0.88	1.04	1.01
Deficit	BART(hs)–o [raw]	0.87	1.02	0.84	**0.85**	1.08	1.10	1.03	1.07	1.17	1.06	0.94	1.12	1.04
Deficit	BART(hs)–o [nl]	0.88	0.99	0.85	0.87	1.07	1.06	0.99	1.06	1.16	1.05	0.94	1.12	1.04
Deficit	BART(hs)–hom [raw]	0.87	1.02	0.85	0.87	1.06	1.06	1.02	1.05	1.11	1.03	0.91	1.06	1.01
Deficit	BART(hs)–hom [nl]	**0.87**	**0.98**	0.84	0.86	1.04	1.02	**0.98**	1.03	1.10	1.02	0.91	1.06	1.01
Debt	BVAR–o	0.91	1.23	0.94	1.04	1.19	**0.94**	**0.86**	**0.81**	**0.78**	**0.85**	**0.70**	**0.57**	**0.49**
Debt	BVAR–hom	2.11	**1.82**	1.63	1.74	**1.93**	1.78	1.61	2.40	2.04	2.08	3.14	3.13	3.34
Debt	BART(proj)–o [raw]	1.41	1.49	1.39	1.50	1.63	1.25	1.43	1.38	1.27	1.29	1.11	0.90	0.86
Debt	BART(proj)–o [nl]	1.13	1.35	1.25	1.23	1.55	1.16	1.31	1.35	1.24	1.26	1.11	0.90	0.86
Debt	BART(proj)–hom [raw]	1.57	1.76	1.50	1.78	1.96	1.30	1.93	1.59	1.27	1.59	1.26	0.89	0.99
Debt	BART(proj)–hom [nl]	1.43	1.66	1.37	1.61	1.85	1.21	1.82	1.55	1.23	1.55	1.26	0.89	0.99
Debt	BART(hs)–o [raw]	0.81	1.25	0.95	0.88	1.41	1.16	1.30	1.24	1.17	1.15	1.02	0.86	0.80
Debt	BART(hs)–o [nl]	**0.74**	1.14	**0.86**	**0.80**	1.30	1.05	1.17	1.19	1.13	1.10	1.02	0.86	0.80
Debt	BART(hs)–hom [raw]	0.93	1.45	1.05	1.01	1.54	1.20	1.42	1.33	1.14	1.21	1.09	0.85	0.83
Debt	BART(hs)–hom [nl]	0.86	1.33	0.95	0.92	1.43	1.09	1.30	1.28	1.10	1.16	1.09	0.85	0.83
RGDP	BVAR–o		0.84	0.89		**0.76**	0.85	0.82	**0.71**	**0.69**	0.73	**0.78**	**0.82**	**0.64**
RGDP	BVAR–hom		0.35	0.33		0.37	0.35	0.39	0.40	0.40	0.43	0.36	0.33	0.47
RGDP	BART(proj)–o [raw]		0.88	0.94		0.80	0.81	0.79	0.74	0.76	0.70	0.84	0.92	0.67
RGDP	BART(proj)–o [nl]		0.85	0.90		0.80	0.82	0.78	0.74	0.76	0.70	0.84	0.92	0.67
RGDP	BART(proj)–hom [raw]		1.02	1.10		0.94	0.93	0.93	0.97	0.97	0.91	1.03	1.19	0.89
RGDP	BART(proj)–hom [nl]		1.01	1.09		0.95	0.93	0.93	0.98	0.98	0.91	1.03	1.19	0.89
RGDP	BART(hs)–o [raw]		0.85	0.89		0.77	**0.76**	0.78	0.73	0.73	0.70	0.84	0.92	0.67
RGDP	BART(hs)–o [nl]		**0.83**	**0.86**		0.76	0.77	**0.76**	0.73	0.73	**0.70**	0.84	0.92	0.67
RGDP	BART(hs)–hom [raw]		0.98	1.04		0.93	0.90	0.90	0.88	0.87	0.88	0.99	1.07	0.84
RGDP	BART(hs)–hom [nl]		0.99	1.04		0.94	0.92	0.91	0.90	0.88	0.89	0.99	1.07	0.84

Table 2: CRPS-R (upside-risk) for quarterly target variables relative (ratios) to the benchmark BVAR-hom (raw predictive losses). Major columns refer to the horizon in quarters, minor columns indicate the month during the quarter the prediction was computed.

Horizon (months)

Variable / Model		CRPS 0	CRPS 1	CRPS 3	CRPS 6	CRPS–L 0	CRPS–L 1	CRPS–L 3	CRPS–L 6	CRPS–R 0	CRPS–R 1	CRPS–R 3	CRPS–R 6
Unemployment	BVAR–o	0.82	0.85	**0.91**	**0.76**	0.81	**0.83**	**0.92**	**0.80**	0.82	0.86	**0.91**	**0.72**
	BVAR–hom	0.30	0.29	0.27	0.32	0.09	0.09	0.08	0.09	0.09	0.09	0.08	0.11
	BART(proj)–o [raw]	0.82	0.86	0.92	0.77	0.82	0.85	0.93	0.81	0.81	0.86	0.91	0.73
	BART(proj)–o [nl]	0.81	0.86	0.92	0.77	0.81	0.84	0.93	0.81	**0.81**	0.86	0.91	0.73
	BART(proj)–hom [raw]	0.88	0.89	0.93	0.79	0.90	0.89	0.92	0.82	0.85	0.88	0.93	0.75
	BART(proj)–hom [nl]	0.84	0.87	0.93	0.79	0.85	0.88	0.92	0.82	0.83	0.87	0.93	0.75
	BART(hs)–o [raw]	**0.81**	**0.85**	0.92	0.77	**0.80**	0.83	0.93	0.81	0.81	0.86	0.91	0.73
	BART(hs)–o [nl]	0.81	0.85	0.92	0.77	0.80	0.84	0.93	0.81	0.81	0.86	0.91	0.73
	BART(hs)–hom [raw]	0.82	0.85	0.92	0.77	0.81	0.83	0.92	0.81	0.82	**0.86**	0.92	0.73
	BART(hs)–hom [nl]	0.81	0.86	0.92	0.77	0.81	0.84	0.92	0.81	0.81	0.86	0.92	0.73
IP	BVAR–o	0.81	**0.66**	**0.74**	**0.60**	0.80	**0.66**	**0.74**	**0.67**	0.82	**0.66**	**0.74**	**0.54**
	BVAR–hom	2.10	2.28	2.07	2.57	0.67	0.74	0.66	0.73	0.65	0.69	0.64	0.90
	BART(proj)–o [raw]	0.84	0.71	0.78	0.64	0.84	0.72	0.78	0.72	0.84	0.70	0.78	0.57
	BART(proj)–o [nl]	0.83	0.71	0.78	0.64	0.83	0.72	0.78	0.72	0.82	0.71	0.78	0.57
	BART(proj)–hom [raw]	1.01	0.90	0.89	0.72	1.01	0.91	0.88	0.78	1.01	0.89	0.90	0.66
	BART(proj)–hom [nl]	0.95	0.87	0.89	0.72	0.95	0.88	0.88	0.78	0.94	0.86	0.90	0.66
	BART(hs)–o [raw]	**0.73**	0.69	0.78	0.64	**0.74**	0.68	0.78	0.71	**0.72**	0.69	0.78	0.57
	BART(hs)–o [nl]	0.74	0.69	0.78	0.64	0.76	0.70	0.78	0.71	0.73	0.69	0.78	0.57
	BART(hs)–hom [raw]	0.83	0.75	0.84	0.68	0.81	0.73	0.83	0.74	0.85	0.76	0.85	0.62
	BART(hs)–hom [nl]	0.82	0.76	0.84	0.68	0.81	0.75	0.83	0.74	0.83	0.76	0.85	0.62
HICP	BVAR–o	0.89	0.86	0.82	0.68	0.90	0.86	0.80	0.59	0.89	0.85	0.83	**0.78**
	BVAR–hom	2.19	2.27	2.37	2.93	0.64	0.65	0.69	0.97	0.72	0.75	0.78	0.86
	BART(proj)–o [raw]	0.81	0.79	**0.77**	**0.67**	0.81	0.78	**0.74**	**0.56**	0.80	0.79	**0.78**	0.78
	BART(proj)–o [nl]	**0.79**	0.78	**0.77**	**0.67**	0.79	0.78	**0.74**	**0.56**	**0.80**	0.78	**0.78**	0.78
	BART(proj)–hom [raw]	0.89	0.85	0.80	0.71	0.92	0.87	0.77	0.59	0.87	0.84	0.82	0.84
	BART(proj)–hom [nl]	0.86	0.83	0.80	0.71	0.86	0.84	0.77	0.59	0.85	0.83	0.82	0.84
	BART(hs)–o [raw]	0.81	0.78	0.78	0.68	0.79	0.77	0.75	0.56	0.82	**0.78**	0.80	0.81
	BART(hs)–o [nl]	0.79	**0.78**	0.78	0.68	**0.78**	**0.77**	0.75	0.56	0.81	0.78	0.80	0.81
	BART(hs)–hom [raw]	0.84	0.82	0.80	0.70	0.83	0.82	0.76	0.58	0.84	0.82	0.82	0.83
	BART(hs)–hom [nl]	0.83	0.81	0.80	0.70	0.81	0.81	0.76	0.58	0.83	0.81	0.82	0.83

Table 3: Variants of CRPS for monthly target variables relative (ratios) to the benchmark BVAR-hom (raw predictive losses). Major columns refer to the horizon in quarters, minor columns indicate the month during the quarter the prediction was computed.

Zooming into relative performances of BART-variants, two aspects are worth noting. First, adding the outlier component improves predictive accuracy in virtually all cases, but not as much as in models with a linear conditional mean. Second, introducing regularization in the context of the linear approximation step (i.e., comparing the proj and hs specifications) improves upon the unrestricted projections used in Huber et al. (2023). Further, we note that using the raw approximation output versus the nonlinearly fitted values for predictions does not materially affect predictive losses.

Conclusions

We compared MF BVAR and BART models to assess their relative predictive performance in forecasting tail risk of a set of quarterly and monthly variables for Italy. Relative to an earlier MF-BART model implementation, we propose an alternative linear approximation step, discuss a computationally efficient estimation algorithm, and consider heteroskedastic errors. We find these econometric refinements to yield improvements in predictive accuracy.

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