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Nonparametric Mixed Frequency Monitoring Macro-at-Risk

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Executive Summary

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Nonparametric Mixed Frequency Monitoring Macro-at-Risk

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July 2025

We compare homoskedastic and heteroskedastic mixed frequency (MF) vector autoregression and Bayesian additive regression tree (BART) models to assess their relative performance in predicting tail risk. MF-BART is a nonlinear state space model, and we discuss linear approximation approaches to devise computationally efficient estimation algorithms. The models are applied in an out-of-sample backcasting, nowcasting and forecasting exercise for a set of quarterly and monthly macroeconomic variables in Italy. The proposed econometric refinements yield improvements in predictive accuracy.

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1. INTRODUCTION

Quantifying macroeconomic risk has received growing attention from academics and policymakers, following the influential work of Adrian *et al.* (2019). Their "growth-at-risk" framework has been applied to other indicators such as inflation or debt (e.g., Adams *et al.*, 2021; Lopez-Salido and Loria, 2024; Furceri *et al.*, 2025). Most related papers use quantile regressions with quarterly data. Some evidence suggests that multivariate models, like vector autoregressions (VARs) with heteroskedastic errors, or nonparametric versions of such models, can improve predictive accuracy (Clark *et al.*, 2023, 2024; Carriero *et al.*, 2024a). Further, using higher-frequency information to predict lower-frequency target variables has proven useful both in single and multiple equation models (e.g., Iacopini *et al.*, 2023; Hauzenberger *et al.*, 2024; Castelnuovo and Mori, 2025).

Motivated by these aspects, we consider parametric and nonparametric versions of a multivariate mixed frequency (MF) model in this paper. As a baseline, we use the MF Bayesian Additive Regression Tree (MF-BART) model of Huber *et al.* (2023), subject to several econometric refinements—indeed, we propose an alternative approximation step for linearizing the underlying nonlinear state space model. Further, we provide a computationally efficient, precision-based, estimation algorithm adapted from Chan *et al.* (2023), which is scalable to high dimensions.

Our empirical work complements Boeck *et al.* (2025), who conduct a forecasting horserace for predicting tail risk in Italy with a similar dataset. By contrast, our focus is on short-horizon forecasts, nowcasts, and backcasts. The latter two are necessary due to the significant publication lags of key variables. And instead of assessing predictions of models estimated separately for quarterly and monthly data, we rely on a joint MF framework in an out-of-sample evaluation exercise. We pick Italy due to its high level of public debt, where risk monitoring is especially important—but the methods are generally applicable. As quarterly targets, we focus on the deficit- and debt-to-GDP ratios, and real GDP. Further, we assess monthly predictions for unemployment, industrial production and inflation. We find MF-VARs equipped with a global-local shrinkage prior and heteroskedastic errors to perform well in predicting upside and downside risk. MF-BART typically exhibits a comparable performance to its linear competitor, although there are noteworthy gains in predictive accuracy for some variables at several horizons. Compared with the MF-BART model of Huber *et al.* (2023) our version offers improvements in predictions.

2. ECONOMETRIC FRAMEWORK

We consider $n_{\mathtt{m}}$ variables $\boldsymbol{y}_{\mathtt{m},t}$ on a monthly frequency, for $t = 1, \dots, T$, and $n_{\mathtt{q}}$ variables $\boldsymbol{y}^{*}_{\mathtt{q},t}$ are observed on a quarterly frequency. We model an associated latent monthly process $\boldsymbol{y}_{\mathtt{q},t}$ that is linked to the observed quarterly variables $\boldsymbol{y}^{*}_{\mathtt{q},t}$ with a set of intertemporal measurement equations. Let $\boldsymbol{y}_t = (\boldsymbol{y}'_{\mathtt{q},t}, \boldsymbol{y}'_{\mathtt{m},t})'$ be an $n \times 1$-vector (i.e., $n = n_{\mathtt{m}} + n_{\mathtt{q}}$), and $\boldsymbol{x}_t = (\boldsymbol{y}'_{t-1}, \dots, \boldsymbol{y}'_{t-p})'$ is of size $k = np$. Following Huber *et al.* (2023), we choose:

$$\boldsymbol{y}_t = \boldsymbol{F}(\boldsymbol{x}_t) + \boldsymbol{u}_t, \quad \boldsymbol{u}_t \sim \mathcal{N}(\boldsymbol{0}_n, \boldsymbol{\Sigma}_t), \tag{1}$$

as state equation, with unknown equation-specific functions $f_i(\boldsymbol{x}_t) : \mathbb{R}^k \to \mathbb{R}$ for $i = 1, \dots, n$, in $\boldsymbol{F}(\boldsymbol{x}_t) = (f_1(\boldsymbol{x}_t), \dots, f_n(\boldsymbol{x}_t))'$. We assume Gaussian errors with a proportionally time-varying covariance matrix $\boldsymbol{\Sigma}_t = o_t \boldsymbol{\Sigma}$. The link between quarterly observed and monthly latent variables is established with $i = 1, \dots, n_{\mathtt{q}}$, measurement equations:

$$y^{*}_{\mathtt{q},it} = \frac{1}{3} y_{\mathtt{q},it} + \frac{2}{3} y_{\mathtt{q},it-1} + y_{\mathtt{q},it-2} + \frac{2}{3} y_{\mathtt{q},it-3} + \frac{1}{3} y_{\mathtt{q},it-4} + \eta_{it}, \quad \eta_{it} \sim \mathcal{N}(0, \omega_i^2), \tag{2}$$

where η_{it} is a measurement error with a small (deterministic) variance ω_i^2, see also Chan *et al.* (2023); (2) is an intertemporal restriction for log-growth rates which exists when we observe a quarterly measure (for $t = 3, 6, 9, \dots$). For other transformations, it is straightforward to adjust this restriction accordingly.

2.1. Approximate sampling of the latent states

We cannot rely on typical methods that are used for sampling the latent states due to the nonlinear conditional mean. Define $\boldsymbol{Y} = (\boldsymbol{y}_1, \ldots, \boldsymbol{y}_T)'$, $\boldsymbol{F}(\boldsymbol{X}) = (\boldsymbol{F}(\boldsymbol{x}_1), \ldots, \boldsymbol{F}(\boldsymbol{x}_T))'$ and $\boldsymbol{U} = (\boldsymbol{u}_1, \ldots, \boldsymbol{u}_T)'$ which are $T \times n$, $\boldsymbol{X} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_T)'$ is $T \times k$, and $\boldsymbol{O} = \text{diag}(o_1^2, \ldots, o_T^2)$, then a full-data version of (1) is:

$$\boldsymbol{Y} = \boldsymbol{F}(\boldsymbol{X}) + \boldsymbol{U}, \quad \text{vec}(\boldsymbol{U}) \sim \mathcal{N}(\boldsymbol{0}_{Tn}, (\boldsymbol{\Sigma} \otimes \boldsymbol{O})). \tag{3}$$

To avoid confusing $\boldsymbol{X}$ as an argument of the function $\boldsymbol{F}(\bullet)$ and linear projections using $\boldsymbol{X}$, we simply write $\boldsymbol{F}(\boldsymbol{X}) = \boldsymbol{F}$ below. We linearly approximate $\boldsymbol{F}$ to update the states, following Huber *et al.* (2023). They use the Moore-Penrose inverse of $\boldsymbol{X}$, $\boldsymbol{X}^+$, to recover a linear approximation $\boldsymbol{F} \approx \boldsymbol{X}\tilde{\boldsymbol{A}}_{\text{proj}}$, where $\tilde{\boldsymbol{A}}_{\text{proj}}$ is a $k \times n$ matrix. If $\boldsymbol{X}$ has full column rank, $\boldsymbol{X}^+ = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'$, and $\tilde{\boldsymbol{A}}_{\text{proj}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{F}$ is the projection of the conditional mean function on $\boldsymbol{X}$.[1] Related aspects are discussed in Crawford *et al.* (2018); Kowal (2022). Huber *et al.* (2023) proceed with $\tilde{\boldsymbol{A}}_{\text{proj}}$ using a filtering/smoothing algorithm, and conditional on draws of the states, sample all other parameters.

For the sake of the argument, suppose $\boldsymbol{F} \approx \boldsymbol{Y}$, i.e., an almost perfect fit (e.g., tree-based implementations tend to overfit the data in-sample), then $\tilde{\boldsymbol{A}}_{\text{proj}}$ will be close to the OLS estimate of the matrix that contains the dynamic VAR coefficients. To reduce noise from fitting unrestricted VARs, we assume independent auxiliary regressions, and introduce approximation errors:[2]

$$f_i(\boldsymbol{x}_t) = d_i + \boldsymbol{x}_t'\tilde{\boldsymbol{a}}_{i,\text{reg}} + e_{it}, \quad e_{it} \overset{\text{iid}}{\sim} \mathcal{N}(0, \varsigma_i^2), \tag{4}$$

[1] Note that we standardize the data before estimation, and transform any quantities back ex post.

[2] This regression does not interact with any other parameters (e.g., it does not affect the prior used for sampling $\boldsymbol{F}$, but is purely used as auxiliary approximation device). Conceptually, the projection-based approach maximizes the R^2 of the linear approximation to $\boldsymbol{F}$, while the proposed approach trades some of this explained variance for a modest amount of approximation error variance by regularizing $\tilde{\boldsymbol{A}}$.

for $i = 1, \ldots, n$, where $\tilde{\boldsymbol{A}}_{\text{reg}} = (\tilde{\boldsymbol{a}}_{1,\text{reg}}, \ldots, \tilde{\boldsymbol{a}}_{n,\text{reg}})$ contains the approximated coefficients associated with the nonlinear conditional mean functions and $\boldsymbol{d} = (d_1, \ldots, d_n)'$, $\boldsymbol{D} = (\boldsymbol{\iota}_T \otimes \boldsymbol{d}')$ contain intercepts. That is, $\boldsymbol{F} = \boldsymbol{X}\tilde{\boldsymbol{A}}_{\text{reg}} + \boldsymbol{D} + \boldsymbol{E}$, $\text{vec}(\boldsymbol{E}) \sim \mathcal{N}(\boldsymbol{0}_{Tn}, \boldsymbol{S} \otimes \boldsymbol{I}_T)$ and $\boldsymbol{S} = \text{diag}(\varsigma_1^2, \ldots, \varsigma_n^2)$. Plugging this into (3), we have $\boldsymbol{Y} = \boldsymbol{X}\tilde{\boldsymbol{A}}_{\text{reg}} + \boldsymbol{D} + \boldsymbol{E} + \boldsymbol{U}$, and, omitting the `reg` subscript (the following also applies to `proj`), obtain:

$$\boldsymbol{y}_t = \boldsymbol{d} + \tilde{\boldsymbol{A}}'\boldsymbol{x}_t + \boldsymbol{\epsilon}_t = \boldsymbol{d} + \sum_{j=1}^{p} \tilde{\boldsymbol{A}}_j \boldsymbol{y}_{t-j} + \boldsymbol{\epsilon}_t, \quad \boldsymbol{\epsilon}_t = \boldsymbol{e}_t + \boldsymbol{u}_t, \tag{5}$$

$\boldsymbol{\epsilon}_t \sim \mathcal{N}(\boldsymbol{0}_n, \boldsymbol{\Sigma}_t + \boldsymbol{S})$, with lag-specific partitions in $\tilde{\boldsymbol{A}}' = (\tilde{\boldsymbol{A}}_1, \ldots, \tilde{\boldsymbol{A}}_p)$ for $j = 1, \ldots, p$, and $\tilde{\boldsymbol{A}}_j$ of size $n \times n$. Next, define $\tilde{\boldsymbol{Y}} = (\boldsymbol{y}_{-p+1}, \ldots, \boldsymbol{y}_0, \boldsymbol{Y}')'$ and $\boldsymbol{y} = \text{vec}(\tilde{\boldsymbol{Y}}')$, $\boldsymbol{m} = \text{vec}(\boldsymbol{D}')$,

$$\boldsymbol{M} = \begin{bmatrix} -\tilde{\boldsymbol{A}}_p & \cdots & -\tilde{\boldsymbol{A}}_1 & \boldsymbol{I}_n & \boldsymbol{0}_{n\times n} & \cdots & \cdots & \boldsymbol{0}_{n\times n} \\ \boldsymbol{0}_{n\times n} & -\tilde{\boldsymbol{A}}_p & \cdots & -\tilde{\boldsymbol{A}}_1 & \boldsymbol{I}_n & \boldsymbol{0}_{n\times n} & \cdots & \boldsymbol{0}_{n\times n} \\ \vdots & \ddots & \ddots & & \ddots & \ddots & \ddots & \vdots \\ \boldsymbol{0}_{n\times n} & \cdots & \boldsymbol{0}_{n\times n} & -\tilde{\boldsymbol{A}}_p & \cdots & -\tilde{\boldsymbol{A}}_1 & \boldsymbol{I}_n & \boldsymbol{0}_{n\times n} \\ \boldsymbol{0}_{n\times n} & \cdots & \cdots & \boldsymbol{0}_{n\times n} & -\tilde{\boldsymbol{A}}_p & \cdots & -\tilde{\boldsymbol{A}}_1 & \boldsymbol{I}_n \end{bmatrix},$$

which allows to rewrite (5):

$$\boldsymbol{M}\boldsymbol{y} = \boldsymbol{m} + \boldsymbol{\nu}, \quad \boldsymbol{\nu} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Xi}), \tag{6}$$

where $\boldsymbol{\Xi} = (\boldsymbol{I}_T \otimes \boldsymbol{S}) + (\boldsymbol{O} \otimes \boldsymbol{\Sigma})$. Further, following Chan *et al.* (2023), let $\boldsymbol{S}_{\mathtt{l}}$ and $\boldsymbol{S}_{\mathtt{o}}$ be selection matrices singling out latent (`l`) and observed (`o`) variables, such that $\boldsymbol{y} = \boldsymbol{S}_{\mathtt{l}}\boldsymbol{y}_{\mathtt{l}} + \boldsymbol{S}_{\mathtt{o}}\boldsymbol{y}_{\mathtt{o}}$. Substituting this expression into (6), we may write $\boldsymbol{M}\boldsymbol{S}_{\mathtt{l}}\boldsymbol{y}_{\mathtt{l}} + \boldsymbol{M}\boldsymbol{S}_{\mathtt{o}}\boldsymbol{y}_{\mathtt{o}} = \boldsymbol{m} + \boldsymbol{\nu}$. Chan *et al.* (2023) show how to derive the (approximate, in our case) distribution of the latent variables conditional on the parameters and observed high frequency variables:

$$\boldsymbol{y}_{\mathtt{l}} | \boldsymbol{y}_{\mathtt{o}}, \bullet \sim \mathcal{N}(\boldsymbol{m}_y, \boldsymbol{K}_y), \tag{7}$$

with moments $\boldsymbol{K}_y = (\boldsymbol{S}_1'\boldsymbol{M}'\Xi^{-1}\boldsymbol{M}\boldsymbol{S}_1)^{-1}$ and $\boldsymbol{m}_y = \boldsymbol{K}_y\boldsymbol{S}_1'\boldsymbol{M}'\Xi^{-1}(\boldsymbol{m} - \boldsymbol{S}_\circ\boldsymbol{y}_\circ)$. Next, we rewrite (2) by stacking across low frequency variables:

$$\boldsymbol{y}_{\mathtt{q}}^* = \boldsymbol{\Lambda}\boldsymbol{y}_1 + \boldsymbol{\eta}_t, \quad \boldsymbol{\eta}_t \sim \mathcal{N}(\mathbf{0}_{n_1}, \boldsymbol{\Omega}), \tag{8}$$

where $\boldsymbol{\Lambda}$ appropriately collects the linear intertemporal restrictions and $\boldsymbol{\Omega}$ the stacked ω_i^2s. We may then combine prior (7) and likelihood (8) to derive the distribution of the states conditional on the quarterly observations: $\boldsymbol{y}_1|\boldsymbol{y}_{\mathtt{q}}^*, \boldsymbol{y}_\circ, \bullet \sim \mathcal{N}(\boldsymbol{m}_y^*, \boldsymbol{K}_y^*)$, with moments $\boldsymbol{K}_y^* = (\boldsymbol{\Lambda}'\boldsymbol{\Omega}^{-1}\boldsymbol{\Lambda} + \boldsymbol{K}_y^{-1})^{-1}$ and $\boldsymbol{m}_y^* = \boldsymbol{K}_y^*(\boldsymbol{\Lambda}'\boldsymbol{\Omega}^{-1}\boldsymbol{y}_{\mathtt{q}}^* + \boldsymbol{K}_y^{-1}\boldsymbol{m}_y)$. Most of these matrices have specific banded structures, or are sparse, and fast computational algorithms can be used—we rely on precision sampling to generate a draw from $p(\boldsymbol{y}_1|\boldsymbol{y}_{\mathtt{q}}^*, \boldsymbol{y}_\circ, \bullet)$, which renders estimation of large systems feasible.

2.2. Model specification, priors and estimation algorithm

The procedure described above yields a draw of the complete history of the data, $\{\boldsymbol{y}_t\}_{t=1}^T$, which we use to estimate the state equation and any other parameters. Indeed, (1) establishes a nonlinear multivariate model (see, e.g., Marcellino and Pfarrhofer, 2024, for a review of the related literature). That is, a Gibbs sampler that iterates between drawing the latent states and all model parameters can be used.

When treating the conditional mean function nonparametrically, we rely on BART as in Chipman *et al.* (2010). We use default priors on the equation-specific functions $f_i(\boldsymbol{x}_t)$ for $i = 1, \ldots, n$. This involves setting priors on splitting variables and thresholds, and the terminal node parameters. Our settings for tuning parameters and the sampling steps are identical to Huber *et al.* (2023). On the constant part of the covariance matrix, we impose a hierarchical inverse Wishart prior to avoid overshrinking, see also Esser *et al.* (2024) and Pfarrhofer and Stelzer (2025) in a static and dynamic multivariate model, respectively. The resulting posterior distributions take standard forms. Further, these papers show how to update all BART-related parameters equation-by-equation, which we do as well. In case we model time-varying variances, we assume that $\Pr(o_t^{1/2} = 1) = 1 - \mathfrak{p}$ and $\Pr(o_t^{1/2} \sim \mathcal{U}(2, \bar{\mathfrak{s}})) = \mathfrak{p}$, where $\mathcal{U}(2, \bar{\mathfrak{s}})$ is a discretely uniform with support $[2, \bar{\mathfrak{s}} = 10]$, and the outlier probability $\mathfrak{p}$ is Beta distributed (see Carriero *et al.*, 2024b).

Finally, in case we use auxiliary regressions for the approximation, we impose tight inverse Gamma priors on $\varsigma_i^2 \sim \mathcal{G}^{-1}(10, 0.01)$. On the coefficients $\text{vec}(\tilde{\boldsymbol{A}}_{\text{reg}}) \sim \mathcal{N}(\mathbf{0}, \lambda \cdot \text{diag}(\tau_1, \ldots, \tau_{nk}))$, we use a horseshoe prior (HS, Carvalho *et al.*, 2010) with a single global parameter λ and local scalings τ_j. This allows for updating the approximate coefficients using textbook results for Bayesian linear regressions. Conditional on a draw of the coefficients, the HS-related parameters are updated using the posteriors provided in Makalic and Schmidt (2015). Note that we implement the same HS when estimating a standard Bayesian VAR (BVAR) assuming a linear functional form, $\boldsymbol{F}(\boldsymbol{x}_t) = \boldsymbol{A}'\boldsymbol{x}_t$.

3. MONITORING TAIL RISK IN ITALY

We conduct an out-of-sample (OOS) exercise to evaluate the predictive accuracy of a set of popular model specifications that are nested in our framework. These competitors are: (1) Bayesian vector autoregression (`BVAR`); (2) `BART` when estimating $\boldsymbol{F}(\boldsymbol{x}_t)$ using BART (Chipman *et al.*, 2010); these are differentiated with respect to how the linearized version is obtained (see Section 2.1): Projection (`proj`), or Horseshoe (`hs`), when relying on (4). Further, we compute and evaluate predictions in two ways: either using the (1) unprocessed (`raw`) output of the linearized sampling step, or (2) fitting values obtained from inserting the linearly approximated states $\boldsymbol{x}_t$ into $\boldsymbol{F}(\boldsymbol{x}_t)$ and adding random shocks, labeled `nl` for "nonlinear," see Huber *et al.* (2023). Both the BVAR and BART versions are estimated with $p = 6$ and either homoskedastic errors (`hom`, $o_t = 1$ for all t), or outlier (`o`) component.

3.1. Dataset and out-of-sample evaluation scheme

Our dataset comprises quarterly and monthly variables from Italy, ranging from January 2001 until June 2024. Specifically, our dataset is patterned after Boeck *et al.* (2025): Deficit-to-GDP (Deficit) and Debt-to-GDP (Debt) ratio, and real GDP growth (RGDP) as quarterly target variables; Unemployment (in differences), industrial production (IP, annualized log-differences), and inflation (HICP, annualized log-differences) as monthly targets.

Further monthly predictors are Italian long-term interest rates (10-year benchmark), the spread between Italian/German long-term government bond yields, economic sentiment indicator, euro area short-term (3-month maturity) rates, and the USD/EUR exchange rate; we refer to Boeck *et al.* (2025) for details. Inspired by the recent work of Furceri *et al.* (2025) who focus on predicting "debt-at-risk," we add several timely indicators of economic/policy uncertainty and financial stress: a geopolitical risk indicator (GPR, Caldara and Iacoviello, 2022, link), composite indicator of systemic stress (CISS, Hollo *et al.*, 2012, ECB data portal) for Italy, and European policy uncertainty (EPU, Baker *et al.*, 2016, link).

The initial training sample uses data from the beginning of the sample until January 2010. Because no history of real-time vintages is available, we truncate the final vintage, such that it respects the release calendar (e.g., in the first month of any quarter, the numbers for the previous two quarters and the current quarter of the debt-ratio have not been released yet; in the second month, only the previous and current quarter are missing). This is reflected in our results, which, on the quarterly frequency, indicate the backcasts as $h \in \{-2, -1\}$, the nowcast (present) as $h = 0$ and the forecasts (future) as $h \geq 1$. Note that in terms of the respective information sets, we assume to compute the predictions at the final day of each month. Our competing models are re-estimated on a monthly basis, adding the most recent available observation for each of the quarterly or monthly indicators. That is, for the quarterly variables we have at least three predictions for the same target quarter (e.g., the $h = 0$ horizon has the 1st, 2nd and 3rd month per quarter).

3.2. Forecasting results

Tables 1 and 2 show quantile-weighted continuous ranked probability scores (CRPSs) as tail forecast metrics. These scoring rules emphasize specific parts of the predictive distribution: downside (left tail, `CRPS-L`) and upside (right tail, `CRPS-R`) risk, see Gneiting and Ranjan (2011). The appendix contains results for the unweighted `CRPS`. The rows of the benchmark, `BVAR-hom` are raw CRPSs (grey shades), all other entries are ratios to this benchmark (blue shading: improvements, red shading vice versa; bold values indicate the best model per variable and horizon). Major columns are horizons (quarters), minor columns indicate months during the quarter. The $h = -2$ column contains only the 1st month indicator, because only “Debt” and “Deficit” are missing (as does the previous quarters’ RGDP for the 3rd month of the 1-step backcast). The predictive losses for monthly forecasts are shown in Table 3, where horizons are on a monthly frequency.

Horizon (quarter / month−per−quarter)

Variable / Model		−2	−1			0			1			2		
		1st	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
Deficit	BVAR−o	**0.78**	1.05	**0.70**	**0.76**	**0.97**	**0.91**	**0.93**	1.09	1.19	**0.91**	1.05	1.07	**0.94**
	BVAR−hom	0.28	**0.31**	0.25	0.26	0.36	0.29	0.32	**0.37**	**0.28**	0.36	**0.43**	**0.37**	0.42
	BART(proj)−o [raw]	0.98	1.07	1.01	1.01	1.10	1.10	0.99	1.20	1.36	1.04	1.09	1.15	1.01
	BART(proj)−o [nl]	0.90	1.03	0.97	0.93	1.09	1.08	0.96	1.19	1.36	1.03	1.09	1.15	1.01
	BART(proj)−hom [raw]	0.98	1.09	1.00	1.10	1.09	1.08	1.12	1.15	1.28	1.12	1.03	1.05	1.03
	BART(proj)−hom [nl]	0.95	1.05	0.96	1.05	1.08	1.05	1.09	1.14	1.27	1.11	1.03	1.05	1.03
	BART(hs)−o [raw]	0.87	1.07	0.89	0.87	1.13	1.07	0.99	1.23	1.34	1.04	1.13	1.15	1.01
	BART(hs)−o [nl]	0.84	1.03	0.88	0.86	1.11	1.02	0.94	1.22	1.33	1.03	1.13	1.15	1.01
	BART(hs)−hom [raw]	0.89	1.05	0.91	0.90	1.05	0.99	0.97	1.14	1.19	1.00	1.05	1.05	0.97
	BART(hs)−hom [nl]	0.86	1.01	0.88	0.88	1.04	0.97	0.94	1.13	1.19	0.99	1.05	1.05	0.97
Debt	BVAR−o	0.93	1.04	0.90	1.05	**0.98**	**0.98**	**0.95**	**0.71**	**0.86**	**0.75**	**0.42**	**0.67**	**0.61**
	BVAR−hom	1.25	**1.23**	1.17	1.16	1.52	1.20	1.18	1.94	1.54	1.70	3.67	1.99	2.20
	BART(proj)−o [raw]	1.23	1.26	1.10	1.24	1.19	1.13	1.20	0.97	0.99	0.93	0.53	0.82	0.76
	BART(proj)−o [nl]	1.11	1.22	1.09	1.14	1.15	1.11	1.16	0.96	0.98	0.93	0.53	0.82	0.76
	BART(proj)−hom [raw]	1.28	1.37	1.17	1.36	1.29	1.24	1.41	1.06	1.09	1.05	0.59	0.87	0.82
	BART(proj)−hom [nl]	1.26	1.34	1.16	1.32	1.25	1.21	1.37	1.05	1.08	1.05	0.59	0.87	0.82
	BART(hs)−o [raw]	**0.85**	1.11	**0.85**	**0.88**	1.06	1.08	1.11	0.88	0.94	0.86	0.50	0.80	0.72
	BART(hs)−o [nl]	0.89	1.09	0.89	0.91	1.02	1.05	1.08	0.87	0.93	0.85	0.50	0.80	0.72
	BART(hs)−hom [raw]	0.95	1.29	0.95	0.94	1.20	1.14	1.18	0.99	0.96	0.89	0.55	0.82	0.74
	BART(hs)−hom [nl]	0.97	1.26	0.97	0.95	1.16	1.10	1.14	0.98	0.96	0.88	0.55	0.82	0.74
RGDP	BVAR−o		**0.80**	**0.83**		**0.85**	**0.84**	**0.79**	**0.77**	**0.74**	**0.82**	**0.70**	**0.73**	**0.69**
	BVAR−hom		0.34	0.33		0.36	0.35	0.34	0.39	0.41	0.39	0.44	0.41	0.45
	BART(proj)−o [raw]		0.90	1.01		0.89	0.91	0.89	0.82	0.81	0.85	0.74	0.82	0.74
	BART(proj)−o [nl]		0.89	0.96		0.90	0.93	0.89	0.82	0.81	0.85	0.74	0.82	0.74
	BART(proj)−hom [raw]		1.03	1.12		0.99	0.98	1.03	0.97	0.96	1.00	0.85	0.98	0.90
	BART(proj)−hom [nl]		1.02	1.09		0.99	0.98	1.02	0.98	0.96	1.00	0.85	0.98	0.90
	BART(hs)−o [raw]		0.85	0.88		0.85	0.84	0.85	0.80	0.76	0.82	0.73	0.79	0.73
	BART(hs)−o [nl]		0.85	0.86		0.86	0.85	0.84	0.80	0.77	0.82	0.73	0.79	0.73
	BART(hs)−hom [raw]		0.93	0.96		0.94	0.92	0.92	0.89	0.85	0.91	0.81	0.87	0.82
	BART(hs)−hom [nl]		0.94	0.97		0.95	0.93	0.93	0.89	0.86	0.92	0.81	0.87	0.82

Table 1: CRPS-L (downside-risk) for quarterly target variables relative (ratios) to the benchmark BVAR-hom (raw predictive losses). Major columns refer to the horizon in quarters, minor columns indicate the month during the quarter the prediction was computed.

Variable	Model	−2	−1			0			1			2		
		1st	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
Deficit	BVAR−o	0.90	1.02	**0.76**	0.94	**0.97**	**0.99**	0.98	**0.95**	1.06	**0.96**	**0.85**	1.01	**0.93**
	BVAR−hom	0.17	0.20	0.16	0.16	0.23	0.18	0.20	0.27	**0.20**	0.23	0.32	**0.24**	0.26
	BART(proj)−o [raw]	0.95	1.03	0.91	0.96	1.04	1.13	1.04	1.00	1.17	1.05	0.89	1.10	1.03
	BART(proj)−o [nl]	0.91	1.00	0.90	0.92	1.04	1.11	1.01	1.00	1.16	1.05	0.89	1.10	1.03
	BART(proj)−hom [raw]	0.98	1.06	0.94	1.03	1.09	1.13	1.11	1.03	1.15	1.09	0.88	1.04	1.01
	BART(proj)−hom [nl]	0.96	1.04	0.92	1.01	1.08	1.10	1.08	1.03	1.14	1.09	0.88	1.04	1.01
	BART(hs)−o [raw]	0.87	1.02	0.84	**0.85**	1.08	1.10	1.03	1.07	1.17	1.06	0.94	1.12	1.04
	BART(hs)−o [nl]	0.88	0.99	0.85	0.87	1.07	1.06	0.99	1.06	1.16	1.05	0.94	1.12	1.04
	BART(hs)−hom [raw]	0.87	1.02	0.85	0.87	1.06	1.06	1.02	1.05	1.11	1.03	0.91	1.06	1.01
	BART(hs)−hom [nl]	**0.87**	**0.98**	0.84	0.86	1.04	1.02	**0.98**	1.03	1.10	1.02	0.91	1.06	1.01
Debt	BVAR−o	0.91	1.23	0.94	1.04	1.19	**0.94**	**0.86**	**0.81**	**0.78**	**0.85**	**0.70**	**0.57**	**0.49**
	BVAR−hom	2.11	**1.82**	1.63	1.74	**1.93**	1.78	1.61	2.40	2.04	2.08	3.14	3.13	3.34
	BART(proj)−o [raw]	1.41	1.49	1.39	1.50	1.63	1.25	1.43	1.38	1.27	1.29	1.11	0.90	0.86
	BART(proj)−o [nl]	1.13	1.35	1.25	1.23	1.55	1.16	1.31	1.35	1.24	1.26	1.11	0.90	0.86
	BART(proj)−hom [raw]	1.57	1.76	1.50	1.78	1.96	1.30	1.93	1.59	1.27	1.59	1.26	0.89	0.99
	BART(proj)−hom [nl]	1.43	1.66	1.37	1.61	1.85	1.21	1.82	1.55	1.23	1.55	1.26	0.89	0.99
	BART(hs)−o [raw]	0.81	1.25	0.95	0.88	1.41	1.16	1.30	1.24	1.17	1.15	1.02	0.86	0.80
	BART(hs)−o [nl]	**0.74**	1.14	**0.86**	**0.80**	1.30	1.05	1.17	1.19	1.13	1.10	1.02	0.86	0.80
	BART(hs)−hom [raw]	0.93	1.45	1.05	1.01	1.54	1.20	1.42	1.33	1.14	1.21	1.09	0.85	0.83
	BART(hs)−hom [nl]	0.86	1.33	0.95	0.92	1.43	1.09	1.30	1.28	1.10	1.16	1.09	0.85	0.83
RGDP	BVAR−o		0.84	0.89		**0.76**	0.85	0.82	**0.71**	**0.69**	0.73	**0.78**	**0.82**	**0.64**
	BVAR−hom		0.35	0.33		0.37	0.35	0.39	0.40	0.40	0.43	0.36	0.33	0.47
	BART(proj)−o [raw]		0.88	0.94		0.80	0.81	0.79	0.74	0.76	0.70	0.84	0.92	0.67
	BART(proj)−o [nl]		0.85	0.90		0.80	0.82	0.78	0.74	0.76	0.70	0.84	0.92	0.67
	BART(proj)−hom [raw]		1.02	1.10		0.94	0.93	0.93	0.97	0.97	0.91	1.03	1.19	0.89
	BART(proj)−hom [nl]		1.01	1.09		0.95	0.93	0.93	0.98	0.98	0.91	1.03	1.19	0.89
	BART(hs)−o [raw]		0.85	0.89		0.77	**0.76**	0.78	0.73	0.73	0.70	0.84	0.92	0.67
	BART(hs)−o [nl]		**0.83**	**0.86**		0.76	0.77	**0.76**	0.73	0.73	**0.70**	0.84	0.92	0.67
	BART(hs)−hom [raw]		0.98	1.04		0.93	0.90	0.90	0.88	0.87	0.88	0.99	1.07	0.84
	BART(hs)−hom [nl]		0.99	1.04		0.94	0.92	0.91	0.90	0.88	0.89	0.99	1.07	0.84

Horizon (quarter / month−per−quarter); Variable / Model

Table 2: CRPS-R (upside-risk) for quarterly target variables relative (ratios) to the benchmark BVAR-hom (raw predictive losses). Major columns refer to the horizon in quarters, minor columns indicate the month during the quarter the prediction was computed.

The BVAR equipped with heteroskedastic errors performs consistently well across variables and horizons; indeed, this is a capable model specification (see, e.g., Carriero *et al.*, 2015, for a related model). Improvements relative to the linear homoskedastic benchmark are sizable in most cases, for both downside and upside risk. `BART(hs)-o` often exhibits comparable losses to its linear heteroskedastic competitor. Although there are noteworthy gains in predictive accuracy for some variables and horizons, it must be acknowledged that especially for backcasts and nowcasts of debt, and selected forecasts of the deficit ratio, the metrics indicate a somewhat weaker performance than the linear model. For nowcasts of the monthly variables, the BART-variants are accurate, and best overall in a large number of cases. The performance for forecasts is on par with `BVAR-o` for unemployment and industrial production, while larger gains arise for inflation, consistent with Clark *et al.* (2023).

Zooming into relative performances of BART-variants, two aspects are worth noting. First, adding the outlier component improves predictive accuracy in virtually all cases, but

Horizon (months)

Variable / Model		CRPS 0	CRPS 1	CRPS 3	CRPS 6	CRPS−L 0	CRPS−L 1	CRPS−L 3	CRPS−L 6	CRPS−R 0	CRPS−R 1	CRPS−R 3	CRPS−R 6
Unemployment	BVAR−o	0.82	0.85	**0.91**	**0.76**	0.81	**0.83**	**0.92**	**0.80**	0.82	0.86	**0.91**	**0.72**
	BVAR−hom	0.30	0.29	0.27	0.32	0.09	0.09	0.08	0.09	0.09	0.09	0.08	0.11
	BART(proj)−o [raw]	0.82	0.86	0.92	0.77	0.82	0.85	0.93	0.81	0.81	0.86	0.91	0.73
	BART(proj)−o [nl]	0.81	0.86	0.92	0.77	0.81	0.84	0.93	0.81	**0.81**	0.86	0.91	0.73
	BART(proj)−hom [raw]	0.88	0.89	0.93	0.79	0.90	0.89	0.92	0.82	0.85	0.88	0.93	0.75
	BART(proj)−hom [nl]	0.84	0.87	0.93	0.79	0.85	0.88	0.92	0.82	0.83	0.87	0.93	0.75
	BART(hs)−o [raw]	**0.81**	**0.85**	0.92	0.77	**0.80**	0.83	0.93	0.81	0.81	0.86	0.91	0.73
	BART(hs)−o [nl]	0.81	0.85	0.92	0.77	0.80	0.84	0.93	0.81	0.81	0.86	0.91	0.73
	BART(hs)−hom [raw]	0.82	0.85	0.92	0.77	0.81	0.83	0.92	0.81	0.82	**0.86**	0.92	0.73
	BART(hs)−hom [nl]	0.81	0.86	0.92	0.77	0.81	0.84	0.92	0.81	0.81	0.86	0.92	0.73
IP	BVAR−o	0.81	**0.66**	**0.74**	**0.60**	0.80	**0.66**	**0.74**	**0.67**	0.82	**0.66**	**0.74**	**0.54**
	BVAR−hom	2.10	2.28	2.07	2.57	0.67	0.74	0.66	0.73	0.65	0.69	0.64	0.90
	BART(proj)−o [raw]	0.84	0.71	0.78	0.64	0.84	0.72	0.78	0.72	0.84	0.70	0.78	0.57
	BART(proj)−o [nl]	0.83	0.71	0.78	0.64	0.83	0.72	0.78	0.72	0.82	0.71	0.78	0.57
	BART(proj)−hom [raw]	1.01	0.90	0.89	0.72	1.01	0.91	0.88	0.78	1.01	0.89	0.90	0.66
	BART(proj)−hom [nl]	0.95	0.87	0.89	0.72	0.95	0.88	0.88	0.78	0.94	0.86	0.90	0.66
	BART(hs)−o [raw]	**0.73**	0.69	0.78	0.64	**0.74**	0.68	0.78	0.71	**0.72**	0.69	0.78	0.57
	BART(hs)−o [nl]	0.74	0.69	0.78	0.64	0.76	0.70	0.78	0.71	0.73	0.69	0.78	0.57
	BART(hs)−hom [raw]	0.83	0.75	0.84	0.68	0.81	0.73	0.83	0.74	0.85	0.76	0.85	0.62
	BART(hs)−hom [nl]	0.82	0.76	0.84	0.68	0.81	0.75	0.83	0.74	0.83	0.76	0.85	0.62
HICP	BVAR−o	0.89	0.86	0.82	0.68	0.90	0.86	0.80	0.59	0.89	0.85	0.83	**0.78**
	BVAR−hom	2.19	2.27	2.37	2.93	0.64	0.65	0.69	0.97	0.72	0.75	0.78	0.86
	BART(proj)−o [raw]	0.81	0.79	**0.77**	**0.67**	0.81	0.78	**0.74**	**0.56**	0.80	0.79	**0.78**	0.78
	BART(proj)−o [nl]	**0.79**	0.78	**0.77**	**0.67**	0.79	0.78	**0.74**	**0.56**	**0.80**	0.78	**0.78**	0.78
	BART(proj)−hom [raw]	0.89	0.85	0.80	0.71	0.92	0.87	0.77	0.59	0.87	0.84	0.82	0.84
	BART(proj)−hom [nl]	0.86	0.83	0.80	0.71	0.86	0.84	0.77	0.59	0.85	0.83	0.82	0.84
	BART(hs)−o [raw]	0.81	0.78	0.78	0.68	0.79	0.77	0.75	0.56	0.82	**0.78**	0.80	0.81
	BART(hs)−o [nl]	0.79	**0.78**	0.78	0.68	**0.78**	**0.77**	0.75	0.56	0.81	0.78	0.80	0.81
	BART(hs)−hom [raw]	0.84	0.82	0.80	0.70	0.83	0.82	0.76	0.58	0.84	0.82	0.82	0.83
	BART(hs)−hom [nl]	0.83	0.81	0.80	0.70	0.81	0.81	0.76	0.58	0.83	0.81	0.82	0.83

Table 3: Variants of CRPS for monthly target variables relative (ratios) to the benchmark BVAR-hom (raw predictive losses). Major columns refer to the horizon in quarters, minor columns indicate the month during the quarter the prediction was computed.

not as much as in models with a linear conditional mean. Second, introducing regularization in the context of the linear approximation step (i.e., comparing the `proj` and `hs` specifications) improves upon the unrestricted projections used in Huber *et al.* (2023). Further, we note that using the raw approximation output versus the nonlinearly fitted values for predictions does not materially affect predictive losses.

4. CONCLUSIONS

We compared MF BVAR and BART models to assess their relative predictive performance in forecasting tail risk of a set of quarterly and monthly variables for Italy. Relative to the MF-BART model proposed in Huber *et al.* (2023), we propose an alternative linear approximation step, discuss a computationally efficient estimation algorithm, and consider heteroskedastic errors. We find these econometric refinements to yield improvements in predictive accuracy.

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Online Appendix: Nonparametric Mixed Frequency Monitoring Macro-at-Risk

A. ADDITIONAL MATERIALS

Figure A.1: Quarterly target variables for the out-of-sample exercise.

Horizon (quarter / month−per−quarter)

Variable / Model		−2	−1			0			1			2		
		1st	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
Deficit	BVAR−o	**0.83**	1.03	**0.73**	**0.84**	**0.97**	**0.95**	**0.96**	1.02	1.12	**0.94**	0.98	1.05	**0.94**
Deficit	BVAR−hom	0.71	0.83	0.67	0.68	0.95	0.76	0.82	**1.03**	**0.78**	0.94	1.20	**0.97**	1.07
Deficit	BART(proj)−o [raw]	0.96	1.05	0.96	0.98	1.07	1.12	1.02	1.10	1.26	1.05	1.01	1.12	1.01
Deficit	BART(proj)−o [nl]	0.91	1.01	0.93	0.92	1.06	1.09	0.98	1.09	1.26	1.04	1.01	1.12	1.01
Deficit	BART(proj)−hom [raw]	0.98	1.07	0.96	1.06	1.08	1.10	1.11	1.09	1.21	1.11	**0.97**	1.04	1.02
Deficit	BART(proj)−hom [nl]	0.96	1.04	0.94	1.02	1.07	1.07	1.08	1.08	1.20	1.10	**0.97**	1.04	1.02
Deficit	BART(hs)−o [raw]	0.87	1.04	0.86	0.86	1.10	1.08	1.00	1.14	1.25	1.05	1.04	1.13	1.02
Deficit	BART(hs)−o [nl]	0.86	1.01	0.86	0.86	1.08	1.04	0.96	1.13	1.24	1.04	1.04	1.13	1.02
Deficit	BART(hs)−hom [raw]	0.88	1.03	0.87	0.88	1.05	1.02	0.99	1.09	1.15	1.02	1.00	1.05	0.98
Deficit	BART(hs)−hom [nl]	0.86	**1.00**	0.86	0.87	1.03	0.99	0.96	1.08	1.14	1.01	1.00	1.05	0.98
Debt	BVAR−o	0.91	1.14	0.91	1.04	1.09	**0.96**	**0.89**	**0.77**	**0.82**	**0.80**	**0.58**	**0.62**	**0.56**
Debt	BVAR−hom	5.37	**4.91**	4.53	4.65	**5.58**	4.79	4.52	6.95	5.73	6.15	10.48	8.07	8.78
Debt	BART(proj)−o [raw]	1.32	1.38	1.25	1.38	1.41	1.20	1.31	1.18	1.14	1.10	0.82	0.87	0.82
Debt	BART(proj)−o [nl]	1.12	1.28	1.17	1.18	1.35	1.14	1.23	1.16	1.12	1.09	0.82	0.87	0.82
Debt	BART(proj)−hom [raw]	1.43	1.57	1.33	1.58	1.62	1.28	1.66	1.33	1.19	1.31	0.93	0.89	0.92
Debt	BART(proj)−hom [nl]	1.35	1.51	1.27	1.47	1.55	1.22	1.59	1.30	1.17	1.29	0.93	0.89	0.92
Debt	BART(hs)−o [raw]	0.82	1.18	0.90	0.88	1.23	1.12	1.20	1.07	1.06	0.99	0.76	0.84	0.77
Debt	BART(hs)−o [nl]	**0.80**	1.11	**0.87**	**0.85**	1.16	1.05	1.11	1.04	1.04	0.97	0.76	0.84	0.77
Debt	BART(hs)−hom [raw]	0.94	1.37	1.00	0.97	1.38	1.17	1.29	1.18	1.06	1.04	0.83	0.84	0.79
Debt	BART(hs)−hom [nl]	0.91	1.30	0.95	0.93	1.31	1.10	1.21	1.15	1.04	1.01	0.83	0.84	0.79
RGDP	BVAR−o		**0.82**	**0.87**		**0.80**	0.83	0.81	**0.73**	**0.70**	0.77	**0.74**	**0.77**	**0.66**
RGDP	BVAR−hom		1.08	1.03		1.16	1.12	1.14	1.25	1.28	1.29	1.23	1.17	1.45
RGDP	BART(proj)−o [raw]		0.89	0.98		0.84	0.85	0.85	0.78	0.77	0.77	0.79	0.86	0.70
RGDP	BART(proj)−o [nl]		0.87	0.94		0.84	0.86	0.83	0.78	0.77	0.77	0.79	0.86	0.70
RGDP	BART(proj)−hom [raw]		1.03	1.11		0.96	0.94	0.98	0.98	0.96	0.95	0.94	1.07	0.89
RGDP	BART(proj)−hom [nl]		1.02	1.09		0.96	0.94	0.98	0.98	0.96	0.95	0.94	1.07	0.89
RGDP	BART(hs)−o [raw]		0.85	0.89		0.80	**0.79**	0.81	0.75	0.73	**0.75**	0.79	0.84	0.69
RGDP	BART(hs)−o [nl]		0.84	0.87		0.80	0.80	**0.81**	0.76	0.73	0.76	0.79	0.84	0.69
RGDP	BART(hs)−hom [raw]		0.96	1.00		0.92	0.90	0.91	0.88	0.85	0.89	0.90	0.96	0.82
RGDP	BART(hs)−hom [nl]		0.97	1.01		0.94	0.92	0.92	0.89	0.86	0.90	0.90	0.96	0.82

Table A.1: CRPS for quarterly target variables relative (ratios) to the benchmark BVAR-hom (raw predictive losses). Major columns refer to the horizon in quarters, minor columns indicate the month during the quarter the prediction was computed.