Appendix 00000

Measuring, Hedging, and Mitigating Climate Risk in Financial Markets and Environmental Resources

Supervisor: Prof. Silvia Romagnoli Speaker: Nicola Bartolini 2nd Workshop on Sustainable Finance GRINS University of Bologna

December 2, 2024

Climate risk and bonds

Appendix 00000

Contents







æ

• • = • • = •

Introduction

The topic of this work is the analysis of the climate risk in financial markets. The climate risk can be divided into two types:

- The physical risk, i.e. the risk associated with financial losses induced by climate events
- The transition risk, is the risk associated with the technological changes induced by the green transition

Appendix 00000

Transition risk variables

- To test the impact of the transition risk we first need to find an appropriate variable to measure it. So, we tested for two variables, the returns of the European Carbon allowances and the transition risk index proposed by Blasberg et al. 2021.
- The choice of European carbon allowances as a transition risk variable is driven by the economic literature in cap and trade system, in equilibrium, is equal to the marginal abatement costs (see Aïd et al. 2023).

Data Summary

- Data Source: Refinitiv
- Commodity:
 - Carbon Emissions (ICE EUA Yearly Energy Future)
- Financial Indices:
 - MSCI Price Index
 - VIX Index
- Stock Dataset: Covers 292 companies across multiple sectors
- CDS Data: CDS data grouped by the emission score

Transition risk and stocks $_{\text{OO} \bullet \text{O}}$

 $\underset{0000}{\text{Climate risk and bonds}}$

References

Appendix 00000

The score driven model for stock log-returns

$$\begin{split} r_{t} &= \mu_{t} + e^{\frac{1}{2}\lambda_{t}}\epsilon_{t} \\ \lambda_{t} &= \omega + \sum_{k=1}^{p} \alpha_{k}s_{t-k} + \sum_{k=1}^{q} \phi_{k}\lambda_{t-k} \\ p(r_{t}|\sigma_{t}^{2}) &= \frac{\exp(-\frac{1}{2}\lambda_{t})}{Beta(\frac{1}{2},\frac{\nu}{2})\sqrt{\nu}} \left(1 + \frac{(r_{t}-\mu_{t})^{2}}{\nu\exp(\lambda_{t})}\right)^{-\frac{\nu+1}{2}} \\ s_{t} &= \frac{1}{2} \left(\frac{(\nu+1)\nu^{-1}r_{t}^{2}\exp(-\lambda_{t})}{1 + \nu^{-1}r_{t}^{2}/\exp(-\lambda_{t})} - 1\right) \\ S_{t} &= \mathcal{I}^{-1} = \frac{2(3+\nu)}{\nu}, \end{split}$$
(1)

æ

イロト イ団ト イヨト イヨト

Regression results

- Investigated dependence between ECF model noise and stock return noise using Kendall-Tau correlation.
- 54% of stocks (159 out of 292) showed a significant Kendall-Tau correlation with ECF.
- Kendall-Tau values were generally positive but low (around 10^{-2}).
- The correlation is likely driven by overall economic activity:
 - Growing economy: higher stock prices and increased demand for Carbon Allowances.
 - Depressed economy: lower stock prices and reduced demand for Carbon Allowances.
- This suggests ECF reflects economic trends, not necessarily transition risk.

Appendix 00000

Climate risk and bonds

In the second part, we tested the impact of climate transition risk and physical climate risk on bond returns. While carbon allowance returns were not statistically significant for bond returns, climate variables were found to significantly impact bond returns, indicating that the market is pricing in physical climate risks. Then the object of the second chapter is the pricing model for including climate variables in bond pricing. Given a probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$, with γ_t as an \mathcal{F}_t -adapted process and τ as the time of default, the price of a

risky zero coupon bond $P_R(0, T)$ with maturity T is expressed as:

$$\begin{split} P_R(0,T) &= P_{RF}(0,T) \mathbb{E}^Q \left[\mathbb{P}(\tau > T | \mathcal{F}_\infty) | \mathcal{F}_0 \right] \\ &+ \delta P_{RF}(0,T) \mathbb{E}^Q \left[\mathbb{P}(\tau \le T | \mathcal{F}_\infty) | \mathcal{F}_0 \right] \\ &= P_{RF}(0,T) \mathbb{E}^Q \left[e^{-\int_0^T \gamma_u du} \Big| \mathcal{F}_0 \right] \\ &+ \delta P_{RF}(0,T) \mathbb{E}^Q \left[1 - e^{-\int_0^T \gamma_u du} \Big| \mathcal{F}_0 \right], \end{split}$$

 γ_t , $t \ge 0$, is the hazard rate, δ is the recovery rate, $P_{RF}(0, T)$ is the risk-free discount factor, and Q is the risk-neutral measure.

The pricing model

The stochastic hazard rate (see Duffie et al. 1999; Driessen 2005) γ_t is expressed as:

$$\gamma_t = \gamma_{0,t} + \beta_1 \gamma_{1,t} + \beta_2 \gamma_{2,t} + \beta_3 \gamma_{3,t} + \beta_4 \gamma_{4,t} + \beta_5 \gamma_{5,t}, \quad (2)$$

Where:

- $\gamma_{0,t}$ is the firm specific factor (C.I.R.),
- Fire Weather Index: $\gamma_{1,t}$ (Lévy OU, exponential jump),
- Average daily Eastward wind speed: γ_{2,t} (Lévy OU, exponential jump),,
- Average daily Northward wind speed: γ_{3,t} (Lévy OU, exponential jump),
- Flood Index: $\gamma_{4,t}$ (C.I.R.),
- Drought Index: $\gamma_{5,t}$ (C.I.R.).

Introduction 00	Transition risk and stocks 0000	Climate risk and bonds 000●	References	Appendix 00000
Results				

- Comparison between the **proposed model** (includes climate variables) and the **alternative model** (only firm-specific risk).
- Objective function means μ_P (proposed) and μ_A (alternative) are compared using a t-test.
- Null hypothesis: $H_0: \mu_P = \mu_A$, Alternative hypothesis: $H_1: \mu_P > \mu_A$ (proposed model performs worse).
- Result: The t-test mostly fails to reject H_0 , indicating similar performance between models across most months.
- Some issuers show evidence that the proposed model outperforms ($H_1: \mu_P < \mu_A$ accepted at 5% or 1% significance).

- 4 同 1 4 三 1 4 三 1

Appendix 00000

- Duffie, Darrell and Kenneth J. Singleton (1999). "Modeling term structures of defaultable bonds". In: *The review of financial studies* 12.4, pp. 687–720.
- Driessen, Joost (2005). "Is default event risk priced in corporate bonds?" In: *The Review of Financial Studies* 18.1, pp. 165–195.
- Benth, Fred Espen, Jūratė Šaltytė Benth, and Steen Koekebakker (2007). "Putting a Price on Temperature*". In: Scandinavian Journal of Statistics 34.4, pp. 746–767.
- Benth, Fred Espen and Jūratė Šaltytė Benth (2011). "Weather derivatives and stochastic modelling of temperature". In:

International Journal of Stochastic Analysis 2011, pp. 1–21.

Härdle, Wolfgang Karl and Brenda López Cabrera (2012). "The implied market price of weather risk". In: *Applied*

Mathematical Finance 19.1, pp. 59–95.



Saltyte Benth, Jūrate and Fred Espen Benth (2012). "A critical view on temperature modelling for application in weather derivatives markets". In: Energy Economics 34.2, pp. 592-602. Creal, Drew, Siem Jan Koopman, and André Lucas (2013). "Generalized autoregressive score models with applications". In: Journal of Applied Econometrics 28.5, pp. 777–795. Harvey, Andrew C (2013). Dynamic models for volatility and heavy tails: with applications to financial and economic time series. Vol. 52. Cambridge University Press. Blasberg, Alexander, Rüdiger Kiesel, and Luca Taschini (2021). "Carbon default swap-disentangling the exposure to carbon risk through CDS". In: Available at SSRN 3856993. Artemova, Marija, Francisco Blasques, Janneke van Brummelen, and Siem Jan Koopman (2022). "Score-driven models: Methodology and theory". In: Oxford Research Encyclopedia of Economics and Finance.

Appendix •0000

- Aïd, René and Sara Biagini (2023). "Optimal dynamic regulation of carbon emissions market". In: *Mathematical Finance* 33.1, pp. 80–115.
- Alfonsi, Aurélien and Nerea Vadillo (2023). A stochastic volatility model for the valuation of temperature derivatives. arXiv: 2209.05918 [q-fin.RM].
- Bartolini, Nicola, Silvia Romagnoli, and Amia Santini (2024).

"A climate risk hedge ? Investigating the exposure of green and non-green corporate bonds to climte risk". In: *Working Paper*.

Dynamics of Risk Indexes

The dynamics of the Fire Weather Index and wind speeds are given by:

$$d\gamma_{i,t} = -k_i\gamma_{i,t} dt + dL^{\mathbb{Q}}_{i,t},$$

$$\gamma_{i,t} = \gamma_{i,t_0}e^{-k_i(t-t_0)} + \int_{t_0}^t e^{-k_i(t-u)}dL^{\mathbb{Q}}_{i,u},$$
(3)

where $L_{i,t}^{\mathbb{Q}}$, with i = 1, 2, 3, are independent compound Poisson processes.

The dynamics of the firm-specific factor, flood, and drought indices are represented by:

$$d\gamma_{i,t} = k_i(\theta_i - \gamma_{i,t}) dt + \sigma_i \sqrt{\gamma_{i,t}} dB^{\mathbb{Q}}_{i,t}, \qquad (4)$$

where $B_{i,t}^{\mathbb{Q}}$, with i = 0, 4, 5, are independent Brownian Motions.

Transition risk and stocks

 $\underset{0000}{\text{Climate risk and bonds}}$

References

Appendix

Pricing Zero Coupon Bonds

For C.I.R. processes, the solution is:

$$\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_{0}^{t}\gamma_{i,u}\,du}\middle|\mathcal{F}_{0}\right] = \exp\{A_{i}(0,t) - C_{i}(0,t)\gamma_{i,t}\},\qquad(5)$$

with components:

$$C_{i}(0,t) = \frac{2(\exp\{td_{i}\}-1)}{2d_{i} + (k_{i} + d_{i})(\exp\{td_{i}\}-1)},$$

$$A_{i}(0,t) = \frac{2k_{i}\theta_{i}}{\sigma_{i}^{2}}\log\left\{\frac{2d_{i}\exp\{(k_{i} + d_{i})t/2\}}{2d_{i} + (k_{i} + d_{i})(\exp\{td_{i}\}-1)}\right\}, \quad (6)$$

$$d_{i} = \sqrt{k_{i}^{2} + 2\sigma_{i}^{2}}.$$

∃ ► < ∃ ►</p>

Transition risk and stocks 0000

 $\underset{0000}{\text{Climate risk and bonds}}$

Appendix 00000

Pricing Zero Coupon Bonds

For Lévy-driven Ornstein-Uhlenbeck processes, the solution is:

$$\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_{0}^{t}\gamma_{i,u}\,du}\middle|\mathcal{F}_{0}\right] = \exp\{H_{i}(0,t) + M_{i}(0,t)\gamma_{i,t}\},\qquad(7)$$

with components:

$$M_{i}(0,t) = \frac{1}{k_{i}} \left(1 - e^{-k_{i}t} \right),$$

$$H_{i}(0,t) = \int_{0}^{t} \lambda_{i} \left(\frac{\eta_{i}}{\eta_{i} + \frac{1}{k_{i}}(1 - e^{-k_{i}(t-s)})} - 1 \right) ds.$$
(8)

• • = • • = •

Estimates climate variables

The results of the fits are in Table 1, for the Ornstein-Uhlenbeck Lévy-driven processes, and in Table 2 for the C.I.R. processes.

	k _i	λ_i	η_i
IT Eastw. Wind	0.6381	1.3748	0.0866
IT Northw. Wind	0.6022	1.2114	0.0929
IT FWI	0.0180	0.0147	0.2280
DE Eastw. Wind	0.4859	1.2501	0.1020
DE Northw. Wind	0.6590	1.6652	0.0846
DE FWI	0.1207	0.0676	0.0817

Table: Calibrated parameters of the Lévy-driven Ornstein-Uhlenbeck processes for weather variables

4 3 5 4 3 5

Transition risk and stocks 0000

Climate risk and bonds

References

Appendix

Estimates climate variables

	ki	θ_i	σ_i
IT Drought	0.0017	0.3256	0.0399
IT Flood	0.0017	0.3623	0.0542
DE Drought	0.0004	0.9938	0.0145
DE Flood	0.0005	0.8482	0.0335

Table: Calibrated parameters of the C.I.R. processes for weather variables

A B M A B M