

An enhanced simulation-based approach for multicriteria evaluation of SMEs' performance

Silvia Angilella^a , Michalis Doumpos^b , Maria Rosaria Pappalardoa,1, Constantin Zopounidis^b

aDepartment of Economics and Business, University of Catania ¹E-mail:maria.pappalardo1@unict.it ^bFinancial Engineering Laboratory, Technical University of Crete

2 nd **Workshop on Sustainable Finance - SPOKE 4 GRINS**

2 *−* 3 **December 2024**

Where we left off...

- *•* **S. Angilella**, M. Doumpos, **M.R. Pappalardo** and C. Zopounidis (2024). *Assessing the performance of banks through an improved sigma-mu multicriteria analysis approach.* Omega, 127, 103099.
- *•* This study provides an extension of the Sigma-Mu efficiency methodology[[Greco et al., 2019\]](#page-33-0) fully accounting for the Pareto dominance relation without eventual inconsistent results in the *σ* − *µ* Pareto-Koopmans global efficiency scores.
- *•* We applied the proposed model to assess the performance of 28 banks from the EU-wide stress tests of EBA on CAMELS and ESG criteria

- *•* Sustainable finance is a theme of growing interest including the definition of augmented credit ratings (ESG criteria).
- *•* The European Commission (EU) has required companies to publish regular reports on the social and environmental risks they face (rules in CSRD).
- *•* Large companies and listed SMEs will now report on sustainability.
- *•* Non listed SMEs are not obliged to report on their sustainability.

Difficulties faced by SMEs for their ESG reporting:

- *•* **Financial constraints:** expenses for consultancy.
- *•* **Lack of expertise:** micro enterprises with limited skills to collect data.
- *•* **Absence of existing data:** data for SMEs are incomplete or missing.
- *•* **Lack of incentive:** sustainability reporting is viewed as a costly burden with unclear benefits.

• Given the increasing focus on SMEs' ESG reporting, methods for evaluating their performance now include ESG criteria.

The aim of this paper is to:

- *•* propose a methodology that combines elements from MCDA with principles of DEA.
- *•* the *σ − µ* efficiency analysis of Greco et al. (2019) and Angilella et al. (2024) is enhanced by incorporating two additional parameters, i.e. skewness (*γ*) and kurtosis (*κ*).
- *•* revise the SMAA model by using the Dirichlet distribution to simulate criteria weights [[Jia et al., 1998\]](#page-33-1), including higher dimensional moments in the SMEs' score.
- *•* evaluate the performance of a sample of European SMEs from 2018-2022 creating a comprehensive composite indicator.

Table of Contents

1. [Methodological background](#page-5-0)

- 1.1 [Preliminary concepts and definitions](#page-6-0)
- 1.2 [Sigma-mu efficiency analysis](#page-7-0)

2. [The proposed methodology](#page-11-0)

- 2.1 [Sampling of the criteria weights](#page-12-0)
- 2.2 [Dominance relation with high order moments](#page-15-0)

3. [Application](#page-18-0)

- 3.1 [Data description](#page-19-0)
- 3.2 [Comprehensive evaluation results](#page-22-0)

4. [Conclusions](#page-27-0)

2

5. [Achievements](#page-29-0)

[Methodological background](#page-5-0)

1.1 Preliminary concepts and definitions

Let $G = \{g_1, g_2, \ldots, g_n\}$ the set of *n* criteria and $|A| = m$ the set of alternatives

Simple aggregation model (Additive value function)¹

$$
V(w, a_x) = \sum_{i=1}^n g_i(a_x) w_i
$$
 (1)

- $g_i(a_x)$ the performance of SME a_x on criterion g_i ;
- *• wⁱ* the weight relative to such criterion; and
- **w** belonging to the unit simplex:

$$
W = \{ (w_1, w_2, \dots, w_n) \in \mathbb{R}^n \colon w_i \ge 0 \text{ and } \sum_{i=1}^n w_i = 1 \}. \tag{2}
$$

 $^{\rm l}$ widely adopted for credit risk assessment and evaluation of financial institutions [\[Tsagkarakis et al., 2021,](#page-34-0) [Doumpos et al., 2017\]](#page-33-2)

- *•* The *σ − µ* analysis[[Greco et al., 2019,](#page-33-0) [Angilella et al., 2024\]](#page-32-0) combines elements from DEA analysis and MCDA.
- *•* Computation of *µ* and *σ* for the alternatives' overall value
- *• σ***-***µ* **Pareto dominance relation**:

$$
a_x D a_y \Leftrightarrow [\mu_x - \mu_y, \sigma_y - \sigma_x] \geq 0, \tag{3}
$$

• [*µ^x − µy, σ^y − σx*] *⇒* each element is no less than zero and at least one element is not zero.

*• σ***-***µ* **Pareto-Koopmans efficiency**: is a stricter concept where an alternative *a^x* is efficient if there is no convex combination $z=(\mu_z,\sigma_z)$ of the remaining alternatives, with

$$
\mu_z = \sum_{j \neq i} \lambda_j \mu_j \quad \text{and} \quad \sigma_z = \sum_{j \neq i} \lambda_j \sigma_j,
$$

such that $z D a_{x}$, where $\lambda_1, \ldots, \lambda_m > 0$ and $\lambda_1 + \cdots + \lambda_m = 1$.

• To verify if *a^x* is *σ*-*µ* Pareto-Koopmans efficient, the following LP problem have to be solved:

$$
\max \delta_x \quad \text{s.t.} \begin{cases} \mu_x \alpha - \sigma_x \beta \ge \mu_y \alpha - \sigma_y \beta + \delta_x \ \forall \ y \ne x \\ \alpha + \beta = 1 \\ \alpha, \beta \ge 0, \delta_x \in \mathbb{R}. \end{cases} \tag{4}
$$

- *•* If a solution exists for the previous LP and *δ^x >* 0, then *a^x* is Pareto-Koopmans efficient.
- *•* In Greco et al. (2019), the local efficiency concept was introduced since an alternative is quite far from the *σ*-*µ* Pareto-Koopmans efficiency frontier.
	- sequence of σ - μ Pareto Koopmans Frontiers (PFKs), denoted by F_1, F_2, \ldots, F_p .
	- *•* Each frontier *F^k* consists of alternatives that are efficient (in terms of the Pareto-Koopmans efficiency concept) compared to the rest of the alternatives, excluding those belonging to "higher" frontiers $F_{k-1} = \{F_1 \cup \cdots \cup F_{k-1}\}.$

• A local σ -*µ* Pareto-Koopmans efficiency score δ_{x} can be defined by solving the following $IP 2$

$$
\max \delta_{xk} \quad \text{s.t.} \begin{cases} \mu_x \alpha - \sigma_x \beta \ge \mu_y \alpha - \sigma_y \beta + \delta_{xk} \ \forall \ y \ne x, y \in \mathcal{P}_k \\ \alpha + \beta = 1 \\ \alpha, \beta \ge 0. \end{cases} \tag{5}
$$

- $\mathcal{P}_{\mathcal{A}} = \mathcal{I} \setminus \mathcal{F}_{k-1}$: set of peers for evaluating the local efficiency of alternative a_x with respect to frontier *k*.
- *•* max solution of *δxk*: *δ ∗ xk*
- \bullet The sum of the local efficiency scores $\delta^*_{x1},\delta^*_{x2},\ldots$ for each alternative is used to derive its global efficiency score $\mathsf{s}_{\mathsf{x}} = \sum_{k} \delta_{\mathsf{x} k}^*$
- *•* Since *s^x* can be negative, a normalized score *s^x ∈* [0*,* 1] can be computed for any alternative as:

$$
\bullet \ \ \overline{s}_x = \tfrac{s_x - \min_x s_x}{\max_x s_x - \min_x s_x}
$$

- *•* **Angilella et al. (2024)** enhance Greco et al. (2019).
- *•* They assess local efficiency differently for the following two cases of alternatives:
	- 1. alternatives from higher-level frontiers that do not dominate any of the remaining alternatives;
		- *• ⇒* **Standard model of***σ***-***µ* **efficiency analysis** [\(5\)](#page-9-0).
	- 2. alternatives not assigned to a higher-level frontier that dominate at least one of the remaining alternatives.
		- *•* a) consider the set of peers against which the performance of an alternative *a^x* is evaluated
		- *•* b) this set of peers is used for comparison for all alternatives that dominate *a^x*
- *•* An alternative *a^y* that dominates *a^x* may also dominate other alternatives at level *k*, thus leading to multiple local efficiency scores ∆*∗ xyk*.
- *•* The final performance score at level *k* is the maximum of all the different results obtained at this level.

[The proposed methodology](#page-11-0)

2.1 Sampling of the criteria weights

- We follow the simulation process of Jia et al. (1998) composed by the following stages:
	- \bullet Choice of the true weights ² (w_1, \ldots, w_n) generated uniformly on the whole weight simplex;
	- We assume that the assessed weights are sampled from a Dirichlet distribution whose single-attribute means correspond to the true weights.
- *•* To generate such weights we generate a set of Gamma variables *dⁱ ∼* Γ(*ωⁱ , αi*):
	- *• ωⁱ* the true weight for attribute *i*;
	- *• αⁱ* a parameter which controls the precision of the assessed weights.
- We normalize weights $w_i = \frac{d_i}{\sum_i y_i}$ *i di* to obtain weights that sum to 1 *⇒* the vector of assessed weights has an m-variate Dirichlet distribution with $\overline{\mathsf{w}_i} = \omega_i;$
- \bullet the Dirichlet parameters $(\alpha_1,\ldots,\alpha_n)$ have as sum: $\alpha=\sum_{i=1}^n\alpha_i.$

² generally unknown.

2.1 Sampling of the criteria weights

The multivariate generalization of the beta distribution

• has the following formula:

$$
f(w_1, w_2, \ldots, w_n) = \frac{\Gamma(\alpha_1 + \alpha_2 + \ldots + \alpha_n)}{\Gamma(\alpha_1) + \Gamma(\alpha_2) + \ldots + \Gamma(\alpha_n)} w_1^{\alpha_1} \cdot w_2^{\alpha_2} \cdot \ldots \cdot w_n^{\alpha_n}.
$$
 (6)

- \bullet The higher α_i , the larger is the weight relative to criterion *i*.
- Generally, parameters are assumed equal $\alpha_1 = \alpha_2 = \ldots = \alpha_n$.
- *•* Three cases:
	- *•* if*α ≈* 1 *⇒*, the Dirichlet distribution is uniform over the simplex*⇒* SMAA [\[Lahdelma et al.,1998](#page-33-3)];
	- *•* setting*α <* 1 *⇒* extreme weights that are closer to corner points of the unit simplex *⇒* some criteria are dictators (MCDA context and not realistic);
- *•* setting*α >* 1 *⇒* more diversified weights concentrated around the center of the simplex approximating the equal weights $\frac{1}{n} \Rightarrow E(\mathbf{w_i}) = \frac{1}{n}$. Workshop on Sustainable Finance - SPOKE 4 GRINS, Venice 12 Canada 12/30 Canada 12/30 Canada 12/30 Canada 12/30

2

nd Workshop on Sustainable Finance - SPOKE 4 GRINS, Venice 13/30

Shapes of beta distribution

We can define 5 scenarios:

2

1. $\alpha_1 = \alpha_2 = 1$: beta distribution collapses to $U \sim (0, 1)$;

2.1 Sampling of the criteria weights

- 2. Left skewed: $\alpha_1 > \alpha_2$ with mean close to 0;
- 3. Right skewed: $\alpha_1 < \alpha_2$ with mean close to 1;
- 4. Platykurtic: $\alpha_1, \alpha_2 < 1$ is symmetric with mean equal to 0*.*5;
- 5. Leptokurtic: $\alpha_1, \alpha_2 > 1$ is symmetric with mean equal to 0*.*5.

Fig. 1: Beta distribution of w_i in different scenarios when $n=2$

• This paper uses a Dirichlet distribution with various parameters (*α*) to model different shapes of stochastic performance score distributions.

Dirichlet distribution with $n = 2$ criteria

2.2 Dominance relation with high order moments

• The starting point of the efficiency analysis implemented are high order moments of the composite indicator:

$$
\mu_{x} = \frac{1}{S} \sum_{r=1}^{S} V(\mathbf{w}_{r}, \mathbf{a}_{x}), \qquad \sigma_{x} = \sqrt{\frac{1}{S} \sum_{r=1}^{S} [V(\mathbf{w}_{r}, \mathbf{a}_{x}) - \mu_{x}]^{2}},
$$

$$
\gamma_{x} = \frac{1}{S} \sum_{r=1}^{S} \left[\frac{V(\mathbf{w}_{r}, \mathbf{a}_{x}) - \mu_{x}}{\sigma_{x}} \right]^{3}, \quad \kappa_{x} = \frac{1}{S} \sum_{r=1}^{S} \left[\frac{V(\mathbf{w}_{r}, \mathbf{a}_{x}) - \mu_{x}}{\sigma_{x}} \right]^{4}, \qquad (7)
$$

- with *S* the number of simulated weights with the Dirichlet distribution (set to 10*,* 000).
- *•* The performance distribution of a composite indicator (formula [1](#page-6-1)) is derived by simulating criteria weights with a Dirichlet distribution.
- *•* Each distribution function of an alternative's score is then described by a quadruplet $(\mu_{\mathsf{x}}, \sigma_{\mathsf{x}}, \gamma_{\mathsf{x}}, \kappa_{\mathsf{x}}).$

2.2 Dominance relation with high order moments

• The Pareto dominance relation *D*, defined in eq. [\(3\)](#page-7-1), can be enhanced including skewness and kurtosis[[Le Courtois and Xu, 2024\]](#page-34-1).

MVSK Model and the dominance relation *R*

$$
MVSK_{x,y} = [\mu_x - \mu_y, \sigma_y - \sigma_x, \gamma_x - \gamma_y, \kappa_y - \kappa_x],
$$

• we can define the dominance relation *R*, in terms of the four moments:

$$
a_x R a_y \Leftrightarrow \text{MVSK}_{x,y} \geq 0, \tag{8}
$$

where each element of *MVSKx,^y* is no less than zero and at least one element is not zero.

2.2 Dominance relation with high order moments

- *•* To assess the local efficiency of *a^x* in terms of the four moments, we distinguish two cases (Angilella et al. (2024)):
	- 1. alternatives from higher-level frontiers that do not dominate any of the remaining alternatives *⇒* **LP 2 is extended by incorporating***γ* **and***κ*,

$$
\max \delta_{xk} \quad \text{s.t.} \begin{cases} \mu_x \alpha - \sigma_x \beta + \gamma_x \theta - \kappa_x \phi \ge \mu_y \alpha - \sigma_y \beta + \gamma_y \theta - \kappa_y \phi + \delta_{xy} \ \forall \ y \ne x, y \in \mathcal{P}_k \\ \alpha, \beta, \theta, \phi \ge 0, \\ \alpha + \beta + \theta + \phi = 1. \end{cases} \tag{9}
$$

2. alternatives not assigned to a higher-level frontier that dominate at least one of the remaining alternatives *⇒* **the set of peers against which the performance of** *a^x* **is evaluated, is also used for comparison for the alternatives that dominate** *a^x* .

• If
$$
(a_x, a_y) \in \mathcal{D} \Rightarrow (a_x, a_y) \in \mathcal{R}
$$

• If(*ax, ^ay*) *∈ R ⇒* (*ax, ^ay*) /*∈ D*

Finanziato
dall'Unione europea
NextGenerationEU

Ň Italiadomani

[Application](#page-18-0)

3.1 Data description

• Dataset: 115 listed European SMEs from Refinitiv database across 2018-2022.

Table 1: Search strategy applied to the Refinitiv database to select the sample of SMEs.

• Due to the Swedish bias (52.17%) in the original sample, we used stratified resampling method, based on the sum of the averages of non-Swedish SMEs per sector.

3.1 Data description

Country distribution after the stratified resampling method

Final sample of SMEs

- *•* SMEs was reduced to **66** after the resampling method
- *•* The residual sample of companies has been further reduced to **46** based on their average number of Employees during 2018-2022
- *•* IRQ method and min-max normalization [\[Gasser et al., 2020\]](#page-33-4), have been utilized to identify outliers, trim data and normalize values based on the following equations:

$$
\bar{g}_i(\mathsf{a}_x) = \frac{g_i(\mathsf{a}_x) - \min_i}{\max_i - \min_i} \quad \text{or} \quad \bar{g}_i(\mathsf{a}_x) = \frac{\max_i - g_i(\mathsf{a}_x)}{\max_i - \min_i}, \quad \text{(10)}
$$

Table 2: Description of the selected ESG criteria and their preference direction.

Table 3: Description of the selected R&D and GROWTH, and FINANCIAL criteria and their preference direction.

3.2 Comprehensive evaluation results

- *•* The MVSK performance score has been computed for each SME.
- We considered three variations of the parameter α (i.e. $\alpha = 3$; $\alpha = 18$; $\alpha = 36$) to conduct a sensitivity analysis on the precision of the assessed weights[[Jia et al., 1998](#page-33-1)].
- *•* Results have been compared with the MV score of Angilella et al. (2024)
- *•* Each step of the methodology has been implemented with a MATLAB code, developed by the authors.

MSVK Results

- *•* PKFs ranges from 8 to 10, with 9 PKFs being the most commonly identified across various years and scenarios;
- *•* This set of frontiers is smaller compared to those obtained with the MV approach of Angilella et al. (2024) (between 12 and 14 frontiers);
- *•* local (*δxk*) and normalized global efficiency scores (*sx*) reveal significant disparities in SME evaluations between the two approaches.

2 *nd* Workshop on Sustainable Finance - SPOKE 4 GRINS, Venice 20/30

 $\mathsf{PKF}_1\text{-}\mathsf{PKF}_2$ in MVSK

3.2 Comprehensive evaluation results

 $\sqrt{ }$ \int

 $\overline{\mathcal{L}}$

Results in terms of PKFs according to the two models (2022, $\alpha = 36$ **).**

- OSE Immunotherapeutics (*a*₂₀),
- *•* Dominion Hosting Holding (*a*32),
- *•* Relief Therapeutics (*a*4),
- Asmallworld (a₈),
- *•* Meriaura Group (*a*12)

*PKF*3-*PKF*9 in MV

 \mathcal{L} $\overline{\mathcal{L}}$

 $\Bigg\}$

3.2 Comprehensive evaluation results

Table 4: Comparison of results between MVSK and MV ranking by year across the three scenarios of *α*.

Fig. 2: Probability distribution of normalized scores with 10,000 simulated weights.

2 *nd* Workshop on Sustainable Finance - SPOKE 4 GRINS, Venice 23/30

Table 5: Pairwise comparison of MVSK and MV rankings for a subset of SMEs in 2022, scenario $\alpha = 36$.

Table 6: Pairwise comparison of MVSK and MV rankings for a subset of SMEs in 2022, scenario $\alpha = 36$.

3.3 Comprehensive evaluation results

Table 7: List of all SMEs' dominance relationships with the MVSK model in 2022, scenario $\alpha = 36$.

Finanziato
dall'Unione europea
NextGenerationEU

ъ.

Ň Italiadomani

[Conclusions](#page-27-0)

4. Conclusions

- *•* We assessed the performances of European SMEs by synthesizing the distribution of composite indicator values, incorporating additional parameters beyond *σ* and *µ*, namely skewness and kurtosis.
- *•* SMAA has been revised by adopting the Dirichlet distribution to the weights of the criteria to capture skewness and kurtosis through the change of some shape parameters.
- *•* Results have been compared with the MV score of Angilella et al. (2024).

MSVK results:

- *•* The set of PKFs obtained with the MSVK is reduced compared to those obtained with the MV approach (8-10 vs 12-14 frontiers).
- *•* Efficiency scores reveal significant disparities in SMEs' evaluations between the two approaches.
- *•* The expansion of evaluation criteria (sigma, mu, skewness and kurtosis) allows for a more comprehensive comparison of alternatives and attenuates their inherent dominance relationship.

Finanziato
dall'Unione europea
NextGenerationEU

[Achievements](#page-29-0)

5. Achievements

• **SECS S/06**

• **S. Angilella**, M. Doumpos, **M.R. Pappalardo** and C. Zopounidis (2024). *Assessing the performance of banks through an improved sigma-mu multicriteria analysis approach.* Omega, 127, 103099.

• **SECS P/11**

- *•* **Galletta, S.**, Mazzù, S., Naciti, V. and Paltrinieri, A. (2024). *A PRISMA systematic review of greenwashing in the banking industry: A call for action.* Research in International Business and Finance, 102262.
- *•* Cosma, S., **Galletta, S.**, Mazzù, S. and Rimo, G. (2024). *Banks' fossil fuel divestment and corporate governance: The role of board gender diversity.* Energy Economics, 139, 107948.
- *•* D'Apolito, E., **Galletta, S.**, Iannuzzi, A. P. and Labini, S.S. (2024). *Sustainability and bank credit access: New evidence from Italian SMEs.* Research in International Business and Finance, 69, 102242

5. Achievements

• **SECS S/01: Antonio Punzo e Roberto Di Mari**

• **Data information**

Units: 1635 listed SMEs, observed for the years 2018-2022. Variables: balance sheet and performance indicators, board characteristics and composition, and ESG disaggregated and aggregated scores.

- *•* **Issues** The final sample has both missing values and (potentially) anomalous observations.
- *•* **Aims**
	- 1. Identifying dynamic clusters of SMEs based on balance sheet and performance characteristics.
	- 2. Assessing how the distinct profiles connect to ESG performance.

• **Methodological solution**

Preliminary step: missings are multiply imputed using random forest techniques. The model: Time-dynamic inhomogeneous hidden Markov model with a fine-tuned outlier-robust emission distribution, extending the OTRIMLE $^{\rm 3}$ approach.

 3 [[Coretto and Hennig, 2016](#page-32-1)]

References

Angilella, S., Doumpos, M., Pappalardo, M. R., & Zopounidis, C. (2024). Assessing the performance of banks through an improved sigma-mu multicriteria analysis approach.

Omega, 127, 103099.

- Angilella, S. and Mazzù, S. (2015). The financing of innovative smes: A multicriteria credit rating model. *European Journal of Operational Research,* 244(2):540–554.
- Angilella, S. and Mazzù, S. (2019). A credit risk model with an automatic override for innovative small and medium-sized enterprises.

Journal of the Operational Research Society, 70(10):1784–1800.

Barro, D., Corazza, M., and Filograsso, G. (2024). Environmental, social, and governance evaluation for European small and medium enterprises: A multicriteria approach,

Corporate Social Responsibility and Environmental Management, 1–18.

Corazza, M., Funari, S., and Gusso, R. (2016). Creditworthiness evaluation of italian smes at the beginning of the 2007–2008 crisis: An mcda approach.

The North American Journal of Economics and Finance, 38:1–26.

Journal of the American Statistical Association, 111(516), 1648-1659. 2 *nd* Workshop on Sustainable Finance - SPOKE 4 GRINS, Venice 28/30

References

Doumpos, M., Hasan, I., & Pasiouras, F. (2017).

Bank overall financial strength: Islamic versus conventional banks.

Economic Modelling, 64, 513-523.

Gasser, P., Suter, J., Cinelli, M., Spada, M., Burgherr, P., Hirschberg, S., ... & Stojadinović, B. (2020). Comprehensive resilience assessment of electricity supply security for 140 countries.

Ecological indicators, 110, 105731.

Greco, S., Ishizaka, A., Tasiou, M., & Torrisi, G. (2019)

Sigma-Mu efficiency analysis: A methodology for evaluating units through composite indicators.

European Journal of Operational Research, 278(3), 942-960.

Jia, J., Fischer, G. W., & Dyer, J. S. (1998). Attribute weighting methods and decision quality in the presence of response error: a simulation study.

Journal of Behavioral Decision Making, 11(2), 85-105.

Lahdelma, R., Hokkanen, J., & Salminen, P. (1998).

SMAA-stochastic multiobjective acceptability analysis.

European Journal of Operational Research, 106(1), 137-143.

References

- Le Courtois, O., & Xu, X. (2024). Efficient portfolios and extreme risks: a Pareto–Dirichlet approach. *Annals of Operations Research,* 335(1), 261-292.
- Roy, P. K. and Shaw, K. (2021). A multicriteria credit scoring model for smes using hybrid bwm and topsis. *Financial Innovation,* 7(1):77.
- H Roy, P. K. and Shaw, K. (2023). A credit scoring model for smes using ahp and topsis.

International Journal of Finance & Economics, 28(1):372–391.

Tsagkarakis, M. P., Doumpos, M., & Pasiouras, F. (2021).

Capital shortfall: A multicriteria decision support system for the identification of weak banks.

Decision Support Systems, 145, 113526.

Voulgaris, F., Doumpos, M., & Zopounidis, C. (2000). On the evaluation of Greek industrial SME's performance via multicriteria analysis of financial ratios.

Small business economics, 15, 127-136.