Tracking-based green portfolio optimization

Diana Barro[1] Marco Corazza[1] Gianni Filograsso[1]

[1] University Ca' Foscari of Venice

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Motivation

- **Sustainable investing** has attracted increasing interest with an associated growing commitment to take an active part in investment choices. Among thematic investments, green and energy-related ones have emerged, capturing investors' attention.
- Standard practice relies on **third-party ESG scores**. Broad evidence points to material disagreements across rating providers, see for example, Billio et al. (2021), Dimson et al. (2020).
- According to a recent survey by Eurosif (2024), asset managers implementing sustainability in their portfolios still rely heavily on **positive and negative screening strategies**.

Integrating Green and ESG preferences

- **Non-optimized strategies** and **traditional portfolio allocation** models cannot guarantee the necessary flexibility.
- Capturing preferences for sustainability is not straightforward. The **attitude of investors** towards green products is **dynamic and heterogeneous**. See, for example, Pastor et al. (2021), Pedersen et al. (2022), Bauer et al. (2021).

Optimally integrating Green and ESG preferences

Integration of green and ESG preferences into portfolio optimization different approaches in the literature:

- Methling and von Nitzsch (2019): **Core-satellite approach** to create tailor-made thematic portfolios.
- Pedersen et al. (2021): **Bi-criterion Sharpe Ratio-ESG** formulation, where the agents preferences affect the shape of the efficient frontier.
- Utz et al. (2015): **A tri-criterion efficient surface**.
- Cesarone et al. (2022): A transformation of the tricriterion problem into a single objective, through ϵ-constraint method.
- Alessandrini et al. (2021): A formulation maximizing the portfolio ESG quality, with constraints on tracking error, exposure to risk factors, concentration risk, sectors and countries.
- ... Several other contributions

Balancing Financial and Environmental Goals

- In this work we focus on a **thematic** approach introduced into a Tracking Error problem.
- The model considers **two tracking goals** that account for different dimensions of the investor's preferences. The overall portfolio is optimized to **jointly track**
  - a **broad market benchmark**
  - and a further **thematic** reference portfolio represented by an **environmental benchmark**.

Scenario-based Double Tracking-Error Approach

The problem is formulated so as to accommodate

- Two different **investment targets**:
  - The **first objective** tracks passively the index, based on Mean Absolute Deviation (MAD), which is referred to as **market component**;
  - The **second objective** tracks a green benchmark with a more active approach, based on downside deviation, which is called **thematic component**;
- **Different preferences** among the two components.
- **Scenario uncertainty** where uncertainty is managed through ML-based scenario generation.

The Portfolio Model

Single-period stochastic optimization problem that offers investors **customizable flexibility** between **profit and sustainability**.

$$\min_{\mathbf{x},\mathbf{y}} \quad \sum_{s=1}^{S} \pi_s \left[\lambda \, \phi_s^M + (1-\lambda)\, \phi_s^G \right]$$

$$\text{s.t.} \sum_{j=1}^{I_E} CI_j y_j \leqslant (1-\eta) \bar{CI}$$

$$\sum_{j=1}^{I_E} E_j y_j \geqslant \bar{E}$$

$$\sum_{i=1}^{I} x_i + \sum_{j=1}^{I_E} y_j = 1$$

$$x_i \geqslant 0 \quad i = 1, \ldots, I \qquad y_j \geqslant 0 \quad j = 1, \ldots, I_E$$

Double Tracking-Error Approach

Scenario-based Double Tracking-Error Portfolio Optimization

- **Objectives**
  - **Passive tracking (MAD)**: $\phi_s^M = \sum_{i=1}^{I} |r_{i,s}x_{i,s} - z_s|$ to mirror the broad financial benchmark
  - **Active tracking (MADD)**: $\phi_s^G = \sum_{i=1}^{I_E} |r_{j,s}y_{j,s} - v_s|^-$ to enhance the result over the green thematic benchmark.
- **Constraints**
  - **Minimum level of greenness** $\sum_{j=1}^{I_E} E_j y_j \geqslant \bar{I}$
  - **Capping carbon intensity** $\sum_{j=1}^{I_E} CI_j y_j < (1-\eta)\bar{CI}$

Linear formulation

$$
\begin{aligned}
\min_{\mathbf{x},\mathbf{y}} \quad & \sum_{s=1}^{S} \pi_s \left[\lambda(\alpha_1 \theta_s^+ + \alpha_2 \theta_s^-) + (1-\lambda)\cdot(\beta_1 \gamma_s^+ + \beta_2 \gamma_s^-) \right] \\
\text{s.t.} \quad & \theta_s^+ - \theta_s^- = \sum_{i=1}^{I} r_{i,s} x_{i,s} - z_s \\
& \gamma_s^+ - \gamma_s^- = \sum_{j=1}^{I_E} r_{j,s} y_{j,s} - w_s \\
& \sum_{j=1}^{I_E} CI_j y_j \leqslant (1-\eta)\overline{CI} \\
& \sum_{i=1}^{I} x_i + \sum_{j=1}^{I_E} y_j = 1 \\
& \gamma_s' - \gamma_s^-, \theta_s^+, \theta_s^-, x_i, y_j \geqslant 0
\end{aligned} \tag{1}
$$

Model specifications

The proposed model is able to represent different levels of preference for environmental commitment. Fixing λ we are able to identify the following 3 models (with $\beta_1 = 0$):

- **Market-Thematic portfolio** ($\lambda = 0.5$) with constraints on carbon intensity;
- **Thematic portfolio** ($\lambda = 0$) with constraints on carbon intensity;
- **Market-decarbonized portfolio** ($\lambda = 1$) with constraints on carbon intensity;
  - Market portfolio ($\lambda = 1$) without constraints on carbon intensity.

GAN Scenario Generation

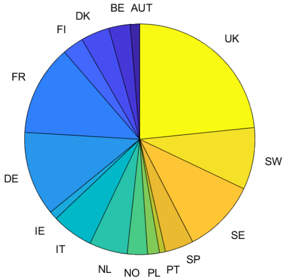
- We adopt a **generative adversarial network (GAN)**, with **Wasserstein-Gradient Penalty (WGAN-GP)** (see, Arjovsky et al. (2017), Gulrajani et al. (2017) and Silva et al. (2023)).
- GANs constitute a recent emerging trend in the literature. The method has the capabilities of handling several stylized facts in financial markets, including temporal dependencies and fat tails of distributions (Dahl et al. (2022)). However, it suffers from hyperparameter sensitivity and computational costs (Takahashi(2019),Koshiyama(2021).

Empirical Results: European Equity Market

- Benchmarks: **EUROSTOXX 600** and **STOXX Europe Env. Leaders**
- Carbon intensity: **GHG Scope 1+2 emissions over Sales** (Bloomberg)

	Mean	Std	Skewness	Kurtosis	Normality	#
EUROSTOXX 600	0.0025	0.0425	-0.4817	4.4050	<0.01	600
STOXX Europe Env. Leaders	0.0034	0.0455	-0.5737	5.3761	<0.001	169

EUROSTOXX 600 contributions



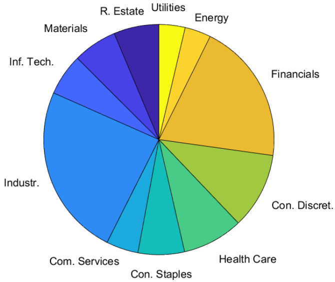


Figure: EUROSTOXX 600 contributions - country and sector dimensions

STOXX Europe Env. Leaders contributions

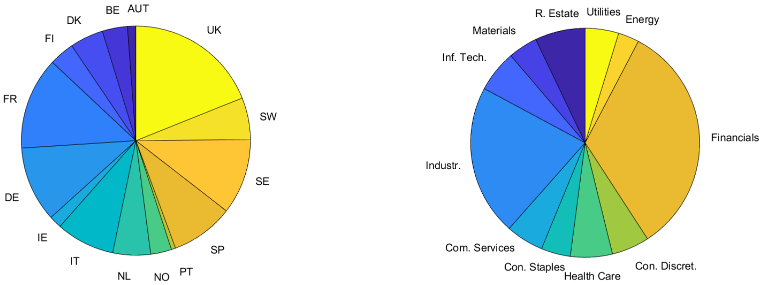


Figure: STOXX Europe Env. Leaders contributions - country and sector dimensions

Empirical applications

- Data: monthly observations from January 31, 2015 to December 29, 2023; two indices, the EUROSTOXX 600 and the STOXX Europe Environmental leaders, along with their constituents, 600 and 169, respectively;
- Scenario generation: GAN-based approach to generate 200 scenarios for each problem. The simulation period runs from January 31, 2015 until December 31, 2022;
- Out-of-sample experiment: one-year investment horizon with a one-month rolling window.

Out-of-sample analysis

	MAE	RMSE	MAPE	$\bar{r}$	σ	SR	OR	$\bar{E}$	$\overline{CI}$
Market	0.0092	0.0105	0.0040	0.0106	0.0400	0.2643	1.8790	2.3660	2507.3
Market-thematic	0.0119	0.0144	0.0069	0.0087	0.0438	0.1977	1.6172	2.4732	1687.7
Market-decarbonized	0.0099	0.0118	0.0042	0.0139	0.0391	0.3550	2.4807	2.4338	2395.3
Thematic	0.0143	0.0164	0.0076	0.0034	0.0373	0.0901	1.2515	2.5243	1558.7

Table: Comparison between out-of-sample error and performance statistics of market, market-thematic, market-decarbonized and thematic portfolios over a 12 months period, from January 2023 to December 2023.

Portfolio composition

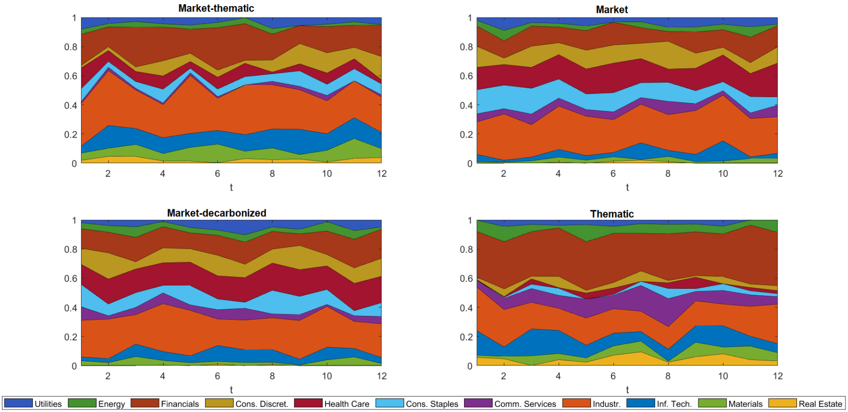


Figure: Breakdown of portfolio holdings by sector for different formulations over time. Top-left: $\lambda = 0.5$, top-right: $\lambda = 1$, bottom-left: $\lambda = 0.5$, bottom-right: $\lambda = 0$.

Portfolio composition

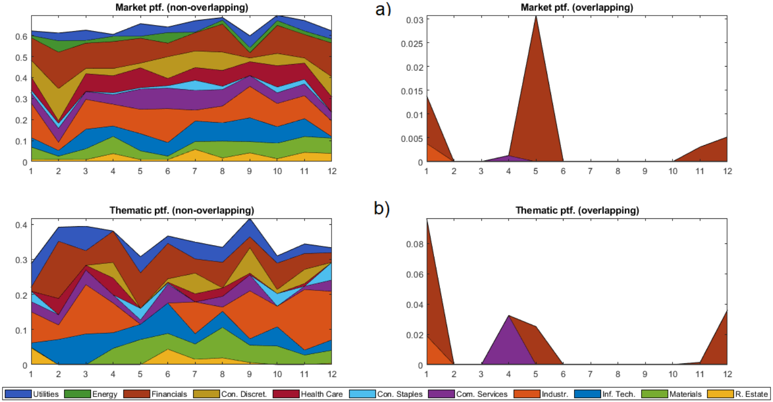


Figure: Breakdown of portfolio compositions contribution by sector, attributed to the Market, Thematic and in the overlapping components, based on an out-of-sample analysis, with $\lambda = 0.5$

Carbon contribution by sector

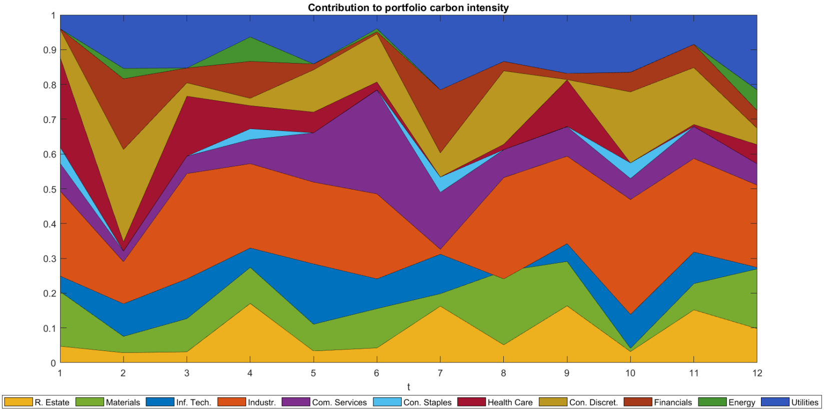


Figure: Breakdown of portfolio carbon intensity contribution by sector, normalized to [0,1] range, based on an out-of-sample analysis over the period January 2023-December 2023, with $\lambda = 0.5$.

Assessing the model

- Stock picking is where the two components significantly diverge;
- A small fraction of the two Market and Thematic components overlaps, with only four sectors involved in the shared portion of the portfolio;
- Stocks that belong to the green benchmark are not crucial for replicating the main index, despite being components of the latter;
- A small subset of assets can replicate adequately any of the two indices, although the number required increases for higher values of λ to achieve higher tracking accuracy;

Assessing the model

- The constraint on carbon intensity is crucial in the sense that more importance is given to the carbon budget of the portfolio;
- A further development considers formulations with different constraints on E performances and carbon emissions;
- The aim is to assess whether additional, stricter constraints on the feasible region have an actual impact on portfolios.

Additional constraints

- A further careful examination of the main statistics of the E scores and carbon intensity shows that high dispersion across firms can be observed.

	Mean	Std	Skew	Kurt	Threshold (I)	Threshold (I_E)
E	2.42	2.11	0.49	2.21	2.42	2.95
CI	1517.08	1115.70	1.18	2.87	1441.12	938.61

Table: Descriptive statistics and constraint thresholds for the E scores and carbon intensity, where I denotes the global sample and I_E the subsample containing the assets within the environmental benchmark.

Formulation

$$
\begin{aligned}
\min_{\mathbf{x},\mathbf{y}} \quad & \sum_{s=1}^{S} \pi_s \left[\lambda(\alpha_1 \theta_s^+ + \alpha_2 \theta_s^-) + (1-\lambda)(\beta_1 \gamma_s^+ + \beta_2 \gamma_s^-) \right] \\
\text{s.t.} \quad & \theta_s^+ - \theta_s^- = \sum_{i=1}^{I} r_{i,s} x_{i,s} - z_s \\
& \gamma_s^+ - \gamma_s^- = \sum_{j=1}^{I_E} r_{j,s} y_{j,s} - w_s \\
& \sum_{j=1}^{I_E} CI_j y_j \leqslant (1-\eta)\overline{CI} \quad \sum_{j=1}^{I_E} E_j y_j \geqslant \bar{E} \\
& \sum_{i=1}^{I} x_i + \sum_{j=1}^{I_E} y_j = 1 \\
& \theta_s^+, \theta_s^-, \gamma_s^+, \gamma_s^-, x_i, y_j \geqslant 0
\end{aligned}
\tag{2}
$$

Metrics

- All the solutions are characterized by similar tracking performance; however difference in environmental performance is remarkable;
- Further analysis to assess the relations among tracking errors, financial measures and the constraints' thresholds

	MAE	RMSE	MAPE	$\bar{r}$	σ	SR	OR	$\bar{E}$	$\overline{CI}$
GHG+E constraint	0.0145	0.0186	0.0067	0.0143	0.0454	0.3145	2.2666	2.2004	1855.3
No constraints	0.0220	0.0250	0.0079	0.0156	0.0524	0.2982	2.2642	1.9164	2469.8
GHG constraint	0.0187	0.0233	0.0085	0.0133	0.0524	0.2543	1.8764	2.1135	1731.1
E constraint	0.0117	0.0143	0.0069	0.0084	0.0440	0.1900	1.5848	2.4677	1971.3

Table: Comparison between out-of-sample error and performance statistics of the market-thematic portfolio assuming different constraints over a 12 months period, from January 2023 to December 2023.

Key Contributions

1. **Flexible Portfolio Optimization**: Our model integrates both financial and ESG benchmarks, allowing investors to tailor portfolios according to their unique risk-return and sustainability preferences.
2. **Carbon Intensity Control and Greeness desired level** : The model includes constraints that allow portfolios to maintain low carbon footprints, aligning with global decarbonization targets and ESG mandates.
3. **Scenario-Based Approach**: By employing Generative Adversarial Networks (GANs) to generate market scenarios, we account for future uncertainties in both financial and thematic performance, ensuring robust portfolio construction.

Future research directions

Definition of a multistage tracking strategy controlling for portfolio rebalancing and associated transaction costs, dealing with the limitations of a static formulation

Thank you!