

WP 4 – Public debt and the financial system under compounding risks



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Short-term Forecasts for the Italian Economy

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Objective

- Provide short term forecasts of key Italian macroeconomic, financial and fiscal indicators:
 - GDP growth;
 - Inflation;
 - Unemployment;
 - Interest rates;
 - Deficit/GDP;
 - Debt/GDP.
- Consider both point and density forecasts;
- Particular attention to tail forecasts to identify and quantify risks in real time.

- Monthly and quarterly datasets from various sources:
 - Central banks: Bank of Italy, ECB, Federal Reserve;
 - Statistical agencies: Istat, Eurostat;
 - Private providers: Revinitiv.
- Bayesian linear and non-linear predictive models:
 - Bayesian VARs;
 - BART;
 - Gaussian Process (GP) regression;
 - Volatility models.

Preview of the results

- Good relative performance of small-size BVAR with Stochastic Volatility for forecasting IP and inflation;
- In general, for horizons up to 1-year ahead:
 - Little gains from non-linear/non-parametric models;
 - Little gains from larger-size models;
- Possibly some advantages of non-parametric in forecasting fiscal variables at longer horizons.

Next step:

- Inspecting quantile scores to assess whether non-linear/non-parametric models offer some gains in predicting tail events.

- We work with general multivariate models of the form:

$$\mathbf{y}_t = F(\mathbf{x}_t) + \epsilon_t \quad (1)$$

\mathbf{y}_t is an $n \times 1$ vector, and $\mathbf{x}_t = (\mathbf{y}'_{t-1}, \dots, \mathbf{y}'_{t-p})'$ is $k = np \times 1$ collecting p lags

- $F(\mathbf{x}_t) = (f_1(\mathbf{x}_t), \dots, f_n(\mathbf{x}_t))'$ are (possibly unknown) conditional mean functions $f_i(\mathbf{x}_t) : \mathbb{R}^k \rightarrow \mathbb{R}$ for $i = 1, \dots, n$, such that $F(\mathbf{x}_t) : \mathbb{R}^k \rightarrow \mathbb{R}^n$
 - ϵ_t reflects the unpredictable component; it is specified such that we can estimate our model equation-by-equation
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- Models will be distinguished with respect to how we treat the function $F(\mathbf{x}_t)$ and what we assume about the error term ϵ_t

Benchmark vector autoregression (VAR)

- The simplest case is to restrict $F(\mathbf{x}_t)$ to linearity, i.e., $F(\mathbf{x}_t) = \mathbf{A}_t \mathbf{x}_t$:

$$\mathbf{y}_t = \mathbf{A}_t \mathbf{x}_t + \epsilon_t$$

\mathbf{A}_t are $n \times k$ (possibly time-varying) VAR coefficients

- VAR: $F(\mathbf{x}_t) = \mathbf{A} \mathbf{x}_t$, constant for all $t = 1, \dots, T$,
- TVP: $F(\mathbf{x}_t) = \mathbf{A}_t \mathbf{x}_t$ combined with state equation for $\mathbf{a}_t = \text{vec}(\mathbf{A}_t)$

$$\mathbf{a}_t = \mathbf{a}_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0_{nk}, \Theta)$$

with a diagonal matrix Θ collecting state innovation variances — these govern the amount of time-variation

Bayesian additive regression trees (BART)

- Sum-of-tree model
- Equation-specific functions $f_i(\mathbf{x}_t)$ are approximated using BART:

$$f_i(\mathbf{x}_t) \approx \sum_{s=1}^S l_{is}(\mathbf{x}_t | \mathcal{T}_{is}, \boldsymbol{\mu}_{is})$$

$l_{is}(\mathbf{x}_t | \mathcal{T}_{is}, \boldsymbol{\mu}_{is})$ are individual tree functions (explaining small fractions)

- Tree structures \mathcal{T}_{is} and terminal nodes $\boldsymbol{\mu}_{is}$ (leaves of the tree)
- BART approximates the function $f_i(\mathbf{x}_t)$ summing over S trees

Gaussian process (GP) regression

- GP prior on the conditional mean functions

$$f_i(\mathbf{x}_t) \sim \mathcal{GP}(0, \mathcal{K}_{\vartheta_i}(\mathbf{x}_t, \mathbf{x}_t))$$

- Using $\mathbf{X} = (\mathbf{x}_1, \dots, \mathbf{x}_T)'$, this prior becomes a (finite) multivariate Gaussian:

$$\mathbf{f}_i \sim \mathcal{N}(0_T, \mathcal{K}_{\vartheta_i}(\mathbf{X}, \mathbf{X}')) , \quad \mathbf{f}_i = (f_i(\mathbf{x}_1), \dots, f_i(\mathbf{x}_T))'$$

- Kernel $\mathcal{K}_{\vartheta_i}(\mathbf{X}, \mathbf{X}')$ with typical (t, \tilde{t}) element $\mathcal{K}_{\vartheta_i}(\mathbf{x}_t, \mathbf{x}_{\tilde{t}})$
 - Distance based, $d_{t\tilde{t}} = \|\mathbf{x}_t - \mathbf{x}_{\tilde{t}}\|^2$ capturing similarities in input space
 - Hyperparameters $\vartheta_i = (\xi_i, l_i)'$ regulate properties
- We use a squared exponential Kernel (others are available)

$$\mathcal{K}_{\vartheta_i}(\mathbf{x}_t, \mathbf{x}_{\tilde{t}}) = \xi_i \times \exp(-l_i d_{t\tilde{t}}/2)$$

Conditional variances

- Few common sources drive reduced form shocks (i.e., primitive shocks)
- Variants of factor stochastic volatility (FSV)

$$\epsilon_t = \mathbf{L}\tilde{\mathfrak{F}}_t + \eta_t$$

$\tilde{\mathfrak{F}}_t$ are $q \times 1$ latent factors linked to observed space by $n \times q$ loadings matrix \mathbf{L} plus idiosyncratic noise $\eta_t = (\eta_{1t}, \dots, \eta_{nt})'$

- We assume heteroskedastic factors

$$\begin{aligned}\tilde{\mathfrak{F}}_t &\sim \mathcal{N}(0_q, \mathbf{\Omega}_t), \quad \eta_t \sim \mathcal{N}(0_n, \mathbf{H}_t), \\ \mathbf{\Omega}_t &= \text{diag}(\exp(\omega_{1t}), \dots, \exp(\omega_{qt})), \\ \mathbf{H}_t &= \text{diag}(\exp(h_{1t}), \dots, \exp(h_{nt}))\end{aligned}$$

- ω_{it} 's and h_{jt} 's follow independent AR(1)

Summary of the models

- General multivariate models of the form:

$$\mathbf{y}_t = F(\mathbf{x}_t) + \mathbf{L}\tilde{\boldsymbol{\varepsilon}}_t + \boldsymbol{\eta}_t, \quad \tilde{\boldsymbol{\varepsilon}}_t \sim \mathcal{N}(0_q, \boldsymbol{\Omega}_t), \quad \boldsymbol{\eta}_t \sim \mathcal{N}(0_n, \mathbf{H}_t)$$

- We consider four options for $F(\mathbf{x}_t) = (f_1(\mathbf{x}_t), \dots, f_n(\mathbf{x}_t))'$
 - ① **Linear/TVP**: $F(\mathbf{x}_t) = \mathbf{A}_t \mathbf{x}_t$, horseshoe shrinkage
 - ② **BART**: $f_i(\mathbf{x}_t) \approx \sum_{s=1}^S l_{is}(\mathbf{x}_t | \mathcal{T}_{is}, \boldsymbol{\mu}_{is})$
 - ③ **GP**: $f_i(\mathbf{x}_t) \sim \mathcal{GP}(0, \mathcal{K}_{\theta_i}(\mathbf{x}_t, \mathbf{x}_t))$
- Homoskedastic by assuming $\mathbf{H}_t = \mathbf{H}$ and $\boldsymbol{\Omega}_t = \mathbf{I}_q$, labeled “hom”
- Heteroskedastic case (stochastic volatility) indicated by “sv,” also versions with t -errors “sv- t ”

Sketch of the estimation algorithm

Standard MCMC algorithm (Gibbs sampling, Metropolis-Hastings steps);
conditional updates for ...

- 1 $F(\mathbf{x}_t)$ equation-by-equation conditional on the factors;
- 2 any hyperparameters related to $F(\mathbf{x}_t)$, e.g., shrinkage for linear/TVP, Kernel hyperparameters for GP, etc.;
- 3 \mathbf{L} and \mathfrak{F}_t conditional on each other, \mathfrak{F}_t in one block;
- 4 log-volatility processes in \mathbf{H}_t and $\mathbf{\Omega}_t$ conditional on $\boldsymbol{\eta}_t$ and \mathbf{f}_t ;
- 5 missing values in \mathbf{y}_t due to the release calendar
- 6 iterative multi-step-ahead forecasts via Monte Carlo to account for all sources of uncertainty, yields \mathbf{y}_{T+h}

Design of the forecasting exercise

- recursive estimation using data from 2000 Q1 / 2000 M9 until hold-out sample
- hold-out sample
 - monthly: 2007M1 - 2022M12
 - quarterly: 2012Q1 - 2022Q4
- two datasets: small (5-7 variables) and large (>10 variables)
- predictive evaluation criteria
 - Mean absolute error (MAE), continuous ranked probability scores (CRPS)
 - reported relative to the benchmark (BVAR-SV)

Predictive loss functions

- Let $y_{i,t+h}^{(r)}$ be the realization and $y_{i,t+h}^{(fp)}$ is the p th quantile of the predictive distribution

- Quantile score (QS)

$$QS(p)_{i,t+h} = 2 \left(y_{i,t+h}^{(r)} - y_{i,t+h}^{(fp)} \right) \times \left(p - \mathbb{I} \left(y_{i,t+h}^{(r)} < y_{i,t+h}^{(fp)} \right) \right),$$

e.g., $QS(0.5)$ is the mean absolute error (MAE, point forecast metric)

- Continuous ranked probability score (CRPS), density forecast metric

$$CRPS_{i,t+h} = \int_0^1 QS(p)_{i,t+h} dp,$$

- Standard Macroeconomic variables: (i) IP, (ii) Hours worked, (iii) Unemployment rate, (iv) HICP, core HICP;
- Survey indicators: expectations on order books in the next three months;
- Monetary policy stance and credit conditions: (i) interest rate on Italian three-months Treasury Bills, (ii) 10y BTP-Bund spread, (iii) Composite Indicator of Systemic Stress (CISS);
- Indicators related to global economic and trade conditions: (i) IP US, (ii) IP Germany, (iii) Export, (iv) Baltic Dry Index.
- Sample period: September 2000 - April 2023.

Monthly variables - Sources and transformations

| Variable | Transformation | Source |
|----------------------------|---------------------|-----------|
| IP excluding construction* | Δ/\log | Istat |
| Hours worked | Δ/\log | Istat |
| Unemployment Rate* | Δ | ECB |
| HICP* | y-o-y Δ/\log | ECB |
| core HICP | y-o-y Δ/\log | ECB |
| Expected order books | level | Istat |
| Treasury Bills rate | Δ | FRED |
| 10y BTP-Bund spread* | level | Refinitiv |
| CISS* | level | ECB |
| IP US | Δ/\log | FRED |
| IP Germany | Δ/\log | Eurostat |
| Export | y-o-y Δ/\log | Istat |
| Baltic Dry Index | \log | Refinitiv |

Notes: Variables denoted by (*) enter the small models.

- Strong in-sample correlation between IP, the survey indicator and the Baltic Dry Index;
- Strong in-sample correlation between inflation, interest rates and the employment indicators;
- Recent years were peculiar:
 - Most extreme values observed for output, hours, export, and the survey indicator belong to the covid-19 period;
 - The highest monthly inflation rate was observed during the energy crises triggered by the Russian invasion of Ukraine;

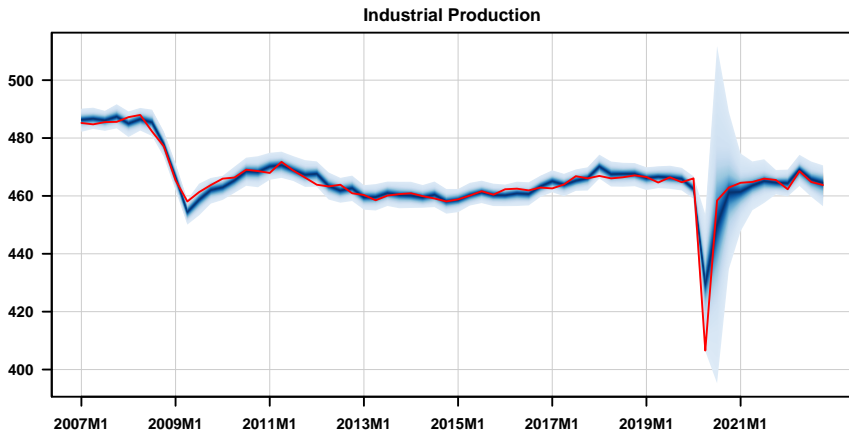
Forecasting Performance - IP

| | | MAE | | | | CRPS | | | |
|-------------|-------------|------|------|------|------|------|------|------|------|
| | | 1 | 3 | 6 | 12 | 1 | 3 | 6 | 12 |
| Small Model | BVAR SV | 2.00 | 1.81 | 1.88 | 1.96 | 1.47 | 1.45 | 1.52 | 1.57 |
| | BVAR SV-t | 1.02 | 1.00 | 0.99 | 0.98 | 1.02 | 0.98 | 0.97 | 0.96 |
| | BVAR TVP-SV | 1.01 | 0.97 | 1.01 | 1.02 | 1.06 | 0.96 | 0.99 | 1.02 |
| | BART hom | 1.03 | 1.06 | 1.07 | 1.01 | 1.07 | 1.03 | 1.04 | 1.01 |
| | BART SV | 0.88 | 1.03 | 1.03 | 1.02 | 0.88 | 1.04 | 1.04 | 1.04 |
| | GP hom | 0.93 | 1.05 | 0.98 | 0.98 | 0.94 | 1.02 | 0.97 | 0.97 |
| | GP SV | 0.89 | 1.02 | 0.99 | 0.96 | 0.86 | 0.99 | 0.97 | 0.95 |
| Large Model | BVAR SV | 1.14 | 1.05 | 1.01 | 0.99 | 1.16 | 1.06 | 1.04 | 1.04 |
| | BVAR SV-t | 1.12 | 1.09 | 1.01 | 0.99 | 1.16 | 1.06 | 1.03 | 1.04 |
| | BVAR TVP-SV | 1.12 | 1.06 | 1.03 | 1.02 | 1.13 | 1.07 | 1.05 | 1.11 |
| | BART hom | 0.91 | 1.08 | 1.03 | 0.99 | 0.94 | 1.05 | 1.00 | 0.98 |
| | BART SV | 0.92 | 1.02 | 1.01 | 0.98 | 0.90 | 1.02 | 1.02 | 1.01 |
| | GP hom | 0.95 | 1.05 | 1.00 | 0.97 | 0.98 | 1.03 | 0.99 | 0.97 |
| | GP SV | 0.95 | 1.04 | 1.00 | 0.98 | 0.93 | 1.03 | 1.01 | 1.02 |

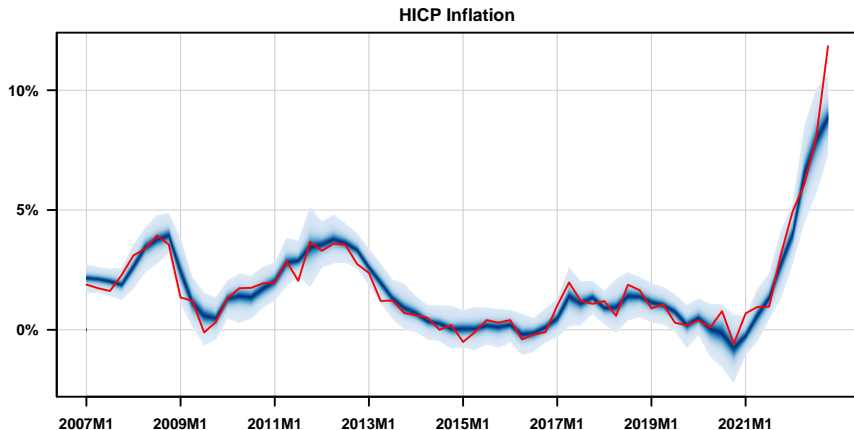
Forecasting Performance - HICP inflation

| | | MAE | | | | CRPS | | | |
|-------------|-------------|------|------|------|------|------|------|------|------|
| | | 1 | 3 | 6 | 12 | 1 | 3 | 6 | 12 |
| Small Model | BVAR SV | 0.36 | 0.71 | 1.02 | 1.42 | 0.27 | 0.55 | 0.78 | 1.12 |
| | BVAR SV-t | 1.01 | 1.02 | 0.99 | 1.01 | 1.01 | 1.02 | 0.98 | 1.01 |
| | BVAR TVP-SV | 1.09 | 0.99 | 0.93 | 1.04 | 1.05 | 0.95 | 0.92 | 1.04 |
| | BART hom | 1.22 | 1.09 | 1.03 | 1.00 | 1.22 | 1.13 | 1.09 | 1.03 |
| | BART SV | 1.43 | 1.16 | 1.11 | 1.03 | 1.41 | 1.21 | 1.16 | 1.06 |
| | GP hom | 1.67 | 1.39 | 1.18 | 1.02 | 1.64 | 1.43 | 1.22 | 1.03 |
| | GP SV | 1.68 | 1.36 | 1.14 | 0.99 | 1.63 | 1.38 | 1.18 | 1.01 |
| Large Model | BVAR SV | 1.04 | 0.98 | 0.98 | 1.02 | 1.05 | 0.95 | 0.95 | 0.99 |
| | BVAR SV-t | 1.05 | 0.99 | 0.99 | 1.06 | 1.06 | 0.97 | 0.98 | 1.05 |
| | BVAR TVP-SV | 1.13 | 0.97 | 1.01 | 1.16 | 1.12 | 0.91 | 0.97 | 1.13 |
| | BART hom | 1.38 | 1.06 | 1.06 | 1.02 | 1.38 | 1.10 | 1.11 | 1.07 |
| | BART SV | 1.55 | 1.30 | 1.11 | 0.98 | 1.55 | 1.33 | 1.16 | 1.03 |
| | GP hom | 2.05 | 1.47 | 1.28 | 1.09 | 2.00 | 1.47 | 1.31 | 1.11 |
| | GP SV | 2.05 | 1.45 | 1.24 | 1.06 | 1.94 | 1.44 | 1.26 | 1.08 |

One-step ahead prediction of industrial production



One-step ahead prediction of HICP inflation



Quarterly variables

- Maastricht deficit variables: (i) government debt (in % of GDP)*, (ii) primary deficit/surplus (in % of GDP)*;
- Macroeconomic variables: (i) GDP*, (ii) GFC, (iii) Trade (IM/EX), (iv) Unemployment rate*, (v) Hours worked, (vi) wages, (vii) labor productivity, (viii) HICP*, (ix) core HICP;
- Survey indicators: consumer sentiment for Italy;
- Monetary policy stance and credit conditions: (i) Euro area short-term interest rate, (ii) Italian long-term interest rates (10y)*, (iii) Composite Indicator of Systemic Stress (CISS)*;
- Indicators related to global economic and trade conditions: (i) IP US, (ii) IP Germany, (iii) IP EA, (iv) EA HICP.
- Sample period: 2000 Q1 - 2023 Q2.

* indicates to be included in the small model

Quarterly variables - Sources and transformations

| Variable | Transformation | Source |
|---|---------------------|--------|
| Government debt* | Δ | ECB |
| Primary deficit/surplus* | Δ | ECB |
| GDP*, GFC, IM, EX, Hours worked | y-o-y Δ/\log | ECB |
| Unemployment rate* | level | ECB |
| Labor productivity, Wages | y-o-y Δ/\log | ECB |
| HICP*, core HICP | Δ/\log | ECB |
| Composite indicator of systemic stress* | level | ECB |
| EA short-term interest rate | level | FRED |
| Long-term interest rate* | level | ECB |
| IP US, IP Germany | y-o-y Δ/\log | FRED |
| IP EA, EA HICP | y-o-y Δ/\log | ECB |
| Consumer sentiment | level | FRED |

Notes: Variables denoted by (*) enter the small models.

- Strong trend behavior in Maastricht deficit variables \Rightarrow taking differences;
- Strong in-sample correlation between domestic and foreign real aggregates;
- Strong in-sample correlation between debt ratio and financing conditions (short- and long-term interest rates);
- Recent years were peculiar:
 - Most extreme values observed for output, hours, export, and the survey indicator belong to the covid-19 period;
 - The highest monthly inflation rate was observed during the energy crises triggered by the Russian invasion of Ukraine;

Forecasting Performance - Government debt

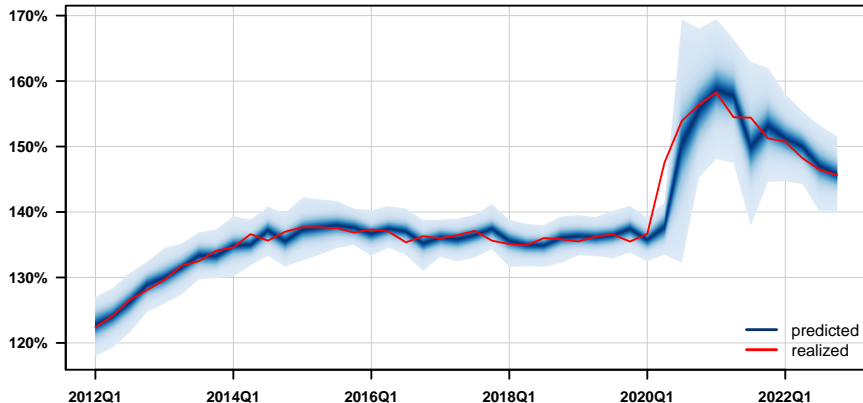
| | | MAE | | | | CRPS | | | |
|-------------|-------------|------|------|------|------|------|------|------|------|
| | | 1 | 4 | 8 | 20 | 1 | 4 | 8 | 20 |
| Small Model | BVAR SV | 1.14 | 3.41 | 5.21 | 8.91 | 0.87 | 2.76 | 4.01 | 5.82 |
| | BVAR SV-t | 1.05 | 1.06 | 1.04 | 1.05 | 1.07 | 1.01 | 0.99 | 1.07 |
| | BVAR TVP-SV | 1.00 | 1.02 | 1.03 | 1.04 | 1.02 | 1.01 | 1.04 | 1.08 |
| | BART hom | 1.15 | 1.16 | 1.10 | 0.97 | 1.16 | 1.14 | 1.10 | 1.04 |
| | BART SV | 1.16 | 1.21 | 1.20 | 1.00 | 1.14 | 1.14 | 1.20 | 1.06 |
| | GP hom | 1.20 | 1.24 | 1.21 | 0.85 | 1.20 | 1.19 | 1.23 | 0.89 |
| | GP SV | 1.25 | 1.24 | 1.19 | 0.88 | 1.23 | 1.19 | 1.21 | 0.92 |
| Large Model | BVAR SV | 1.14 | 1.06 | 1.21 | 1.17 | 1.14 | 1.03 | 1.16 | 1.20 |
| | BVAR SV-t | 1.33 | 1.32 | 1.31 | 1.37 | 1.40 | 1.25 | 1.26 | 1.52 |
| | BVAR TVP-SV | 1.29 | 1.32 | 1.53 | 2.35 | 1.37 | 1.22 | 1.45 | 2.59 |
| | BART hom | 1.23 | 1.28 | 1.35 | 0.80 | 1.22 | 1.25 | 1.34 | 0.84 |
| | BART SV | 1.22 | 1.25 | 1.35 | 0.82 | 1.17 | 1.19 | 1.32 | 0.85 |
| | GP hom | 1.28 | 1.33 | 1.37 | 0.76 | 1.25 | 1.27 | 1.34 | 0.78 |
| | GP SV | 1.27 | 1.33 | 1.38 | 0.77 | 1.24 | 1.26 | 1.34 | 0.78 |

Forecasting Performance - Government deficit

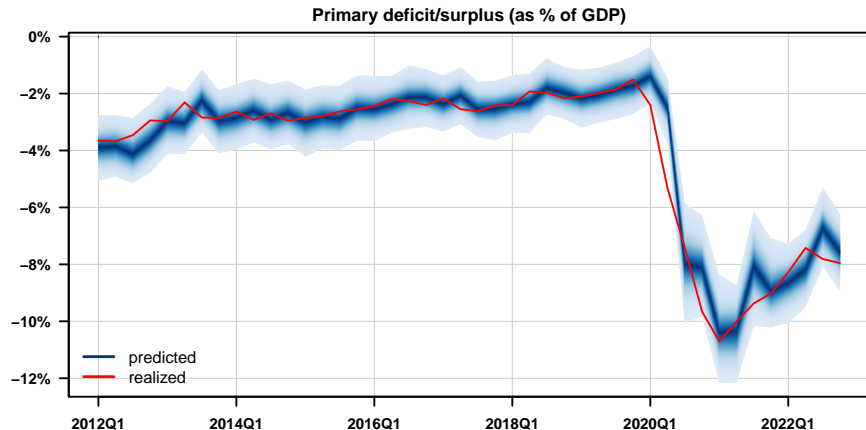
| | | MAE | | | | CRPS | | | |
|-------------|-------------|------|------|------|------|------|------|------|------|
| | | 1 | 4 | 8 | 20 | 1 | 4 | 8 | 20 |
| Small Model | BVAR SV | 0.43 | 1.36 | 2.24 | 3.80 | 0.31 | 1.12 | 1.93 | 3.11 |
| | BVAR SV-t | 1.02 | 1.02 | 1.02 | 0.98 | 1.03 | 1.03 | 1.00 | 0.96 |
| | BVAR TVP-SV | 1.00 | 1.03 | 1.03 | 1.02 | 1.01 | 1.02 | 1.02 | 1.00 |
| | BART hom | 1.00 | 0.98 | 0.87 | 0.88 | 1.08 | 1.00 | 0.91 | 0.89 |
| | BART SV | 0.98 | 0.98 | 0.90 | 0.89 | 1.01 | 0.97 | 0.92 | 0.87 |
| | GP hom | 1.05 | 0.99 | 0.86 | 0.88 | 1.14 | 0.99 | 0.89 | 0.90 |
| | GP SV | 1.04 | 1.00 | 0.87 | 0.87 | 1.11 | 0.98 | 0.90 | 0.89 |
| Large Model | BVAR SV | 0.91 | 0.98 | 1.03 | 0.98 | 0.94 | 0.99 | 1.02 | 0.98 |
| | BVAR SV-t | 0.94 | 1.00 | 1.12 | 1.17 | 0.98 | 1.02 | 1.10 | 1.10 |
| | BVAR TVP-SV | 0.98 | 1.09 | 1.24 | 1.16 | 0.95 | 1.08 | 1.17 | 1.20 |
| | BART hom | 1.04 | 1.01 | 0.87 | 0.87 | 1.11 | 1.02 | 0.89 | 0.85 |
| | BART SV | 1.01 | 0.99 | 0.88 | 0.87 | 1.03 | 0.97 | 0.90 | 0.83 |
| | GP hom | 1.04 | 0.95 | 0.84 | 0.84 | 1.12 | 0.97 | 0.88 | 0.86 |
| | GP SV | 1.03 | 0.95 | 0.84 | 0.83 | 1.08 | 0.95 | 0.89 | 0.85 |

One-step ahead prediction of government debt

Government debt (as % of GDP)



One-step ahead prediction of government deficit



Conclusion and Way Forward

Main Takeaways:

- BVAR-SV model performs overall quite good
- Non-linear models offer some gains for single variables (monthly IP) or longer horizons (quarterly government debt/deficit)
- perspective so far focused on mean predictions

Next Steps:

- finalization of information set
- investigation of tail behavior through non-linear approaches
- evaluation of quantile scores

Thank You!